Foreclosures and House Price Dynamics:
A Quantitative Analysis of the Mortgage Crisis and the 
Foreclosure Prevention Policy

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Abstract

We construct a quantitative equilibrium model of the housing market in which an unanticipated increase in the supply of housing triggers default mortgages via its effect on house prices. The decline in house prices creates an incentive to increase the consumption of housing space but leverage makes it costly for homeowners to sell their homes and buy bigger ones (they must absorb large capital losses). Instead, leveraged households find it advantageous to default and rent housing space. Since renters demand less housing space than homeowners, foreclosures are a negative force affecting house prices. We explore the possible effects of the government’s foreclosure prevention policy in our model. We find that the policy can temporarily reduce foreclosures and shore up house prices.

Key Words: Leverage, Foreclosures, Mortgage Crisis, Loan Modifications, Foreclosure Prevention Policy

JEL:
1 Introduction

The goal of this project is to construct a quantitative equilibrium model of the housing market in which a shock to fundamentals triggers default via its effect on house prices. Our analysis is motivated by the recent experience of declining house prices and rising defaults on mortgages in the United States (and elsewhere) and the US government’s attempt to stabilize the housing market by instituting a program of loan modifications to prevent foreclosures. We use our quantitative model to explore the possible impacts of the foreclosure prevention policy.

The key elements of our model environment are as follows. We imagine a city with an exogenously given but potentially time-varying stock of housing. The city is populated by a continuum of infinitely-lived residents subject to uninsured idiosyncratic shocks to earnings. Residents can buy consumption goods, save in the form of a risk-free savings account with an exogenously given and constant interest rate, and purchase or rent their housing space. If a resident chooses to purchase housing space, he or she can borrow funds from a mortgage market to do so. Residents must pay tax on earnings and interest income. As in the US federal tax code, interest payments on mortgages can be deducted from taxable income and implicit rental income from owner-occupancy is excluded from taxable income.

We model the mortgage market as competitive, with every borrower being charged an interest rate that exactly reflects the borrower’s objective probability of default. A key determinant of this default risk is the level of downpayment made on the loan – a level that we assume is freely chosen by the borrower. The endogeneity of the downpayment will imply that when the risk of default is perceived to be low, the downpayment chosen will be low as well and homeowners will be highly leveraged.

We locate the proximate cause of the mortgage crisis in "overbuilding": an increase in the supply of housing that fails to be matched by an increase in demand at the going price. In equilibrium, the price of housing must fall to absorb the increase in supply. Our model shows how leverage and the tax benefits of homeownership can conspire to turn a simple adjustment of demand and supply into a crisis. Leverage implies that when the price of housing falls selling one’s home imposes a capital loss. Instead of selling their homes and buying a bigger one, the capital loss of a sale induces homeowners to give up their homes in foreclosure and rent for a while. But renting does not have the same tax benefits as homeownership so the erstwhile homeowners tend to rent less space than they previously owned. In this fashion, foreclosures and renting become forces contributing to less housing demand, not more. Consequently foreclosures end up exacerbating the downward adjustment of house prices.

With regard to foreclosure prevention policy we find that the policy being pursued by the government can be effective in decreasing foreclosures and thereby shoring up house prices. However, our analysis also reveals that the expiration of the policy may be followed by another crash in housing prices and another spike in foreclosures. The reason is simply that by delaying foreclosures the policy delays adjustment to the higher supply of housing. Thus, once the policy expires the adjustment starts anew and home prices fall again.
2 Literature Review


3 Environment

We will study the housing equilibrium in a representative city. Time is discrete and indexed by $t = 0, 1, 2, \ldots$
The city has a fluctuating stock of housing space $H(t) > 0$.

3.1 People

There is a fixed continuum of individuals. Individuals derive utility from the consumption of a homogenous consumption good and the service flow from housing space. We permit the service flow from owner-occupied housing space to be potentially higher than the the service flow from rented housing space. Let $c_t$ denote consumption of the homogenous good in period $t$, let $h_t$ denote the consumption of housing space in period $t$ and let $e_t$ be an indicator variable that is 1 if the person owns the housing space $h_t$ and 0 otherwise. Then an individual values the consumption stream

$$U(c, h, e) = \sum_{t=0}^{\infty} \beta^t [u(c_t, h_t) + \nu \cdot e_t], 0 < \beta < 1, \nu \geq 0.$$  

We assume that

$$u(c_t, h_t, 0) = \frac{[c_t^{1-\theta} h_t^\theta]^{1-\gamma}/(1-\gamma)}{[c_t^{1-\theta} h_t^\theta]^{1-\gamma}/(1-\gamma)}.$$  

Note that housing space used by a resident must be either fully owned or fully rented.

Each resident independently draws a earnings level $w$ from a finite-state Markov process with non-negative positive support $W \subset R_+$. The probability that $w_{t+1} = w'$ given $w_t = w$ is $F(w', w)$.

3.2 Market Arrangement

The homogeneous consumption/endowment good is the numeraire good. There are four markets in this economy.

1. There is a market for owner-occupied housing in which the price per unit of housing space in period $t$ is $P(t)$. An individual who buys $k'$ units of housing pays a purchase price of $P(t) \cdot k'$. We assume that owner-occupied housing comes in discrete sizes given by the finite set $K$. 

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2. Second, there is a market for rental housing in which the rent per unit of housing space in period \( t \) is \( R(t) \). An individual who rents \( h \) units of housing space in period \( t \) pays \( R(t) \cdot h \) as rent.

3. Third, there is a market for risk-free deposits. The interest rate on deposits is \( r > 0 \). An individual who saves \( a \) units receives \( a(1 + r) \) next period.

4. Finally, there is a market for mortgages where individuals can borrow to fully or partially fund the purchase of a house. For tractability we assume that all mortgages are perpetuities – in the sense that each mortgagor pays some agreed to amount \( x \) each period unless the mortgagor defaults or the mortgage is paid off. In case of default, the financial intermediary gets ownership of \( h \). In case the mortgage is paid off, the lender receives present value of the promised payment \( x \) discounted at the risk-free rate \( r \). Because of the possibility of default, the (unit) price \( q \) of a mortgage depends on the amount of housing pledged as collateral \( h \), the payment amount \( x \) promised in perpetuity, the individual’s post-purchase savings \( y \), his current earnings \( w \), and the time period \( t \). An individual who takes out a mortgage in period \( t \) with perpetuity payment \( x \) receives \( q(x, h, y, w, t) \cdot x \) in the current period.

### 3.3 Government Sector

There is also a government sector that levies taxes on income. For simplicity, we assume that government consumption of goods does not have provide any benefits to households. The amount of taxes \( g \) to be paid by an individual is modeled after the US tax code. An individual’s tax liability depends on the individual’s taxable income which is calculated as the sum of earnings and interest income minus the greater of (i) mortgage payment \( x \) or (ii) the standard deduction \( s \). That is:

\[
g(w, a, x) = \tau(w + ra - \max[x, s]) \cdot (w + ra - \max[x, s]). \tag{3}
\]

Here the tax rate, or bracket, \( \tau(\cdot) \) is weakly increasing in taxable income.

### 3.4 Financial Intermediaries

Financial intermediaries take in deposits, sell mortgages, and own the housing space rented by people. All intermediaries can borrow or lend funds in a world credit market at a given risk-free interest rate \( \bar{r} > 0 \). We will assume that there is one representative risk-neutral intermediary that takes all prices as given.
4 Decision Problems

4.1 People

The state variables for individuals are $k, x, a, w$ and whether the individual is excluded from the mortgage market or not. Consider first the decision problem of an individual who does not own housing and who is not excluded from the mortgage market due to prior default. For this person $k = 0$ and $x = 0$ and the person may choose to rent or she may choose to buy. Since the person is not excluded from the mortgage market she can borrow to purchase a house. In this case, if the individual chooses to purchase, she solves:

$$M_1(0, 0, a, w; t) = \max_{c\geq 0, k' \in K, x \geq 0, y \geq 0} \{u(c, k') + \nu + \beta E_{w' | w} V(k', x, y, w', t + 1)\}$$

$$c = w - g(w, a, x) + a(1 + r) - y - P(t)[1 + \chi_B]k' + q(x, k', y, w, t) \cdot x$$

If the individual is excluded from the mortgage market due to a prior default but chooses to purchase a house, she solves:

$$M_X^1(0, 0, a, w; S) = \max_{c\geq 0, k' \in K, y \geq 0} \{U(c, k') + \nu + \beta E_{w' | w} \lambda V(k', 0, y, w', t + 1) + (1 - \lambda)X(k', 0, y, w', t + 1)\}$$

$$c = w - g(w, a, x) + a(1 + r) - y - P(t)[1 + \chi_B]k'$$

If the individual is not excluded from the mortgage market and chooses to rent she solves

$$M_0(0, 0, a, w; t) = \max_{c\geq 0, h \geq 0} \{U(c, h) + \beta E_{w' | w} V(0, 0, y, w', t + 1)\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y - R(t) \cdot h$$

If the individual is excluded from the mortgage market and chooses to rent she solves

$$M^X_0(0, 0, a, w; S) = \max_{c\geq 0, h \geq 0} \{U(c, h) + \beta E_{w' | w} \lambda V(0, 0, y, w', t + 1) + (1 - \lambda)X(0, 0, y, w', t + 1)\}$$

$$c = w - g(w, a, 0) + a(1 + r) - y - R(S) \cdot h$$

Then

$$V(0, 0, a, w; t) = \max \{M_1(0, 0, a, w; t), M_0(0, 0, a, w; t)\}$$

And

$$X(0, 0, a, w; t) = \max \{M^X_1(0, 0, a, w; t), M^X_0(0, 0, a, w; t)\}$$

Consider next the decision problem of an individual who owns a house and has an outstanding mortgage. The household may choose to keep the current house, sell it or default on the mortgage. If she chose to keep
the house, she solves:

\[ K_1(k, x, a, w; t) = \max_{c \geq 0, y \geq 0} \{ U(c, k) + \nu + \beta E_{w' | u} V(k, x, y, w', t + 1) \} \]
\[ c = w - g(w, a, x) + a(1 + r) - y - x \]

If he chooses to sell he solves:

\[ K_0(k, x, a, w; t) = \max_{c \geq 0, h \geq 0, s \geq 0} \{ U(c, h) + \beta E_{w' | u} V(0, 0, y, w', t + 1) \} \]
\[ c = w - g(w, a, 0) + a(1 + r) - y - P(t)[1 - \chi_S]k - (1 + 1/\bar{r}) \cdot x - R(t) \cdot h \]

Here \( \chi_S \) is the cost of selling a house and is some percentage of the sale price of the house \( P(t) \). Selling the house requires the individual to buy back the perpetuity \( x \) at the risk-free interest rate.\(^1\) If the household chooses to default on the mortgage, he solves:

\[ K_D(k, x, a, w; S) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{ U(c, h) + \beta E_{w'} V(0, 0, y, w', t + 1) + (1 - \lambda)X(0, 0, a', w', t + 1) \} \]
\[ c = w - g(w, a, 0) + a(1 + r) - y - \chi_D - R(t) \cdot h \]

Foreclosure results in the individual losing the house as well as the mortgage and being excluded from the mortgage market for some random length of time.

An individual who is a homeowner but excluded from the mortgage market can either keep her house or sell it. If she chooses to keep her house she solves:

\[ K^X_1(k, 0, a, w; t) = \max_{c \geq 0, a' \geq 0} \{ U(c, k) + \nu + \beta E_{w' | u} V(k, 0, y, w', t + 1) + (1 - \lambda)X(k, 0, y, w', t + 1) \} \]
\[ c = w - g(w, a, x) + a(1 + r) - y \]

If she chooses to sell she solves:

\[ K^X_0(k, 0, a, w; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{ U(c, h) + \beta E_{w'} V(0, 0, y, w', t + 1) + (1 - \lambda)X(0, 0, a', w', t + 1) \} \]
\[ c = w - g(w, a, 0) + a(1 + r) - y - P(t)[1 - \chi_S]k - R(t) \cdot h \]

Then for \( k > 0 \) and \( x \geq 0 \)

\[ V(k, x, a, w, t) = \max \{ K_0(k, x, a, w; t), K_1(k, x, a, w; t), K_D(k, x, a, w; t) \} \]

And for \( k > 0 \)

\[ X(k, 0, a, w, t) = \max \{ K^X_0(k, 0, a, w; t), K^X_1(k, 0, a, w; t) \} \]

For non-excluded households, let \( c(k, x, a, w, t) \), \( k'(k, x, a, w, t) \), \( x'(k, x, a, w, t) \), \( h(k, x, a, w, t) \), \( a(k, x, a, w, t) \) and \( d(k, x, a, w, t) \) denote the optimal decision rules for consumption, size of owned housing, size of mortgage

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\(^1\)Ideally we should require the individual to buy back the perpetuity at \( \tilde{q} \cdot x \) where \( \tilde{q} \) is the price the individual paid when he issued the perpetuity. But to implement this would require expanding the state vector.
payments, housing services, financial asset sand default (\(d = 1\) indicates default and \(d = 0\) indicates no default). Also, let \(s(k, x, a, w, t) = 1\) indicate the case where the individual sells the house, i.e., \(s(k, x, a, w, t) = 1\) if and only if \(k > 0\), \(d(k, x, a, w, t) = 0\), and \(k'(k, x, a, w, t) = 0\). And let \(c^X(k, 0, a, w, t), k'^X(k, 0, a, w, t), h^X(k, 0, a, w, t), a^X(k, x, a, w, t)\) and \(s^X(k, 0, a, w, t)\) be the decisions of households who are excluded from the mortgage market.

4.2 Financial Intermediaries

The representative financial intermediary accepts deposits, buys mortgages, and buys and rents housing. We will assume that the intermediary chooses how much to engage in these activities on the basis of the expected return in each of these activities. Denote the intermediary’s expected net rate return (profits) on a deposit of size \(a\) by \(\pi(a)\), the net expected return on a mortgage with characteristics \(k', x', y, w\) and \(t\) by \(\pi(k', x', y, w, t)\) and the net expected return on the purchase of \(h\) units of housing in period \(t\) by \(\pi(h, t)\). Correspondingly, let \(m(a)\), \(m(k', x', y, w, t)\) and \(m(h, t)\) denote the measure (more precisely, the density or pdf) of such contracts acquired by the financial intermediary. Then, the financial intermediary’s decision problem is:

\[
\Pi(t) = \max_{\{m(a), m(k', x', y, w, t), m(h, t)\}} \left\{ \int \pi(a)m(da) + \int \pi(k', x', y, w, t)m(dk', dx', dy, dw, t) + \int \pi(h, t)m(dh, t) \right\}
\]

For there to be a (bounded) solution to this problem the net expected returns on each type of asset must be non-positive. For deposits this requires \(\pi(a) = a - [a(1 + r)]/(1 + \bar{r}) \leq 0\) for all \(a\). This requirement reduces to

\[
(1 + r) \geq (1 + \bar{r}). \tag{4}
\]

For housing, this requires that \(\pi(h, t) = R(t) \cdot h + [P(t + 1) \cdot (1 - \delta)h]/(1 + \bar{r}) - P(t)h \leq 0\) for all \(h\) (financial intermediaries do not pay any cost for selling houses), where \(\delta\) is the depreciation on rental properties. This requirement reduces to

\[
P(t) \geq R(t) + [P(t + 1)(1 - \delta)]/(1 + \bar{r}) \tag{5}
\]

For mortgages, the expression for net return is more involved. When the intermediary acquires a mortgage it gives up \(q(k', x, y, w, t) \cdot x\) in goods. Next period, if the individual defaults the intermediary receives \(P(t + 1)[1 - \chi_L D]k'\) where \(\chi_L D\) is the cost of foreclosure to the intermediary; if the individual sells the property the intermediary receives \(x \cdot (1 + 1/\bar{r})\); and if she neither defaults nor sells then the intermediary receives \(x\) plus the value of the continuing mortgage which is then given by \(q(k', x, a(k', x, y, w', t + 1), w', t + 1) \cdot x\). Recalling the definition of \(d(k, x, a, w, t)\) and \(s(k, x, a, w, t)\), the requirement that the expected net return from a mortgage \(\pi(k', x', y, w, t)\) be non-positive becomes
\[ q(k', x', y, w, t) x' \geq (1 + r)^{-1} \times \\
E_{w|w'} \{ d(k', x', y, w', t + 1) P(t + 1) [1 - \chi_{LD}] k' + s(k', x', y, w', t + 1) [x'(1 + 1/\bar{r})] + \\
(1 - d(k', x', y, w', t + 1))(1 - s(k', x', y, w', t + 1)) q(k', x', a(k', x', y, w', t + 1), w', t + 1) \cdot x' \} \]

5 Equilibrium

An equilibrium consists of a sequence of rental pricing function \( R^*(t) \), a sequence of housing price functions \( P^*(t) \), a deposit interest rates \( r^* \), a sequence of mortgage price functions \( q^*(k, x, a, w, t) \), a sequence of distributions \( \mu^*(k, x, a, w, t) \) of people over the state space, and a sequence of decision rules for excluded and non-excluded individuals such that:

1. The decision rules are optimal given \( r^*, R^*(t), P^*(t), q^*(t) \).
2. The net returns (4)-(6) are zero.
3. Demand for housing equals supply, that is, \( \int h^*(k, x, a, w, t) \mu^*(dk, dx, da, dw; t) = H(t) \)
4. The sequence of distributions \( \mu^*(k, x, a, w, t) \) are implied by the decision rules.

6 Parameter Selection and Calibration

Turning first to the Markov process for earnings, we assume that log earnings follows an AR1 process:

\[ \ln(w_t) = \rho \ln(w_{t-1}) + \epsilon_t \]

Several studies have estimated this process for the US using PSID earnings data. Estimates of \( \rho \) and the standard deviation of \( \epsilon \) (\( \sigma_\epsilon \)) vary across studies. We follow Jeske and Krueger (2005) and set \( \rho = 0.95 \) and \( \sigma_\epsilon = 0.129 \). This AR1 process is then approximated by a 5-state Markov chain.

Setting aside the parameters of the income tax schedule, our model economy has 11 other parameters. These include 4 preference parameters (\( \beta, \theta, \gamma, \nu \)), 3 parameters related to housing transactions (\( \chi_S, \chi_B, \delta \)), 3 related to costs of foreclosures (\( \lambda, \chi_D, \chi_{LD} \)) and 1 related to financial markets (\( r \)).

For some parameters we use information from microstudies to restrict their values. Among preference parameters, we set \( \theta = 0.24 \) based on the findings in Davis and Ortao-Magne (2009) that across US cities with quite different real rental prices of housing, expenditure on rent as percentage of household income is remarkable close to 0.24. Among the housing transactions parameters, we appeal to Gruber and Martin (2003) to set \( \chi_B = 0.025 \) and \( \chi_S = 0.075 \) and among the foreclosure cost parameters, we set \( \chi_{LD} = 0.22 \) on the basis of the findings reported in Pennington-Cross (2006). The real interest rate is set at 4 percent.
The remaining 6 parameters are determined by computing the steady state of the model economy for a given housing stock and setting these parameters so as to match certain data targets. In this version of the paper, we have set both $\nu$ and $\chi_D$ to 0. The remaining 4 parameters are chosen to come as close as possible to the (i) the average home-ownership rate over the period 1993-2003 (ii) average fraction of mortgages going into foreclosure over the period 1993-2003, (iii) the ratio of mean housing wealth to mean financial wealth in 1998 (the midpoint of the period 1993-2003), (iv) the ratio of the median household net worth to median household income in 1998. We chose to target averages over the period 1993-2003 so as not to distort our parameter choices by booms and busts in the residential real estate markets. These targets are displayed in Table 1.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>homeowners/total households</td>
<td>0.664</td>
<td>Orzechowski and Sepielli (2003) Table A, p. 5</td>
</tr>
<tr>
<td>foreclosure initiation/outstanding mortgages</td>
<td>0.0056</td>
<td>Mortgage Bankers Assoc., Guler (2008)</td>
</tr>
<tr>
<td>mean housing wealth/mean household net worth</td>
<td>0.337</td>
<td>O. &amp; S. Table A, p. 5</td>
</tr>
<tr>
<td>median hh net worth/median hh income</td>
<td>1.28</td>
<td>O. &amp; S. Table C, p. 8 &amp; Table H-8 (Cen.B.)</td>
</tr>
</tbody>
</table>

The home-ownership rate may be computed as the fraction of households who are homeowners or as the fraction of housing units that are owner-occupied. The number reported in the table is the former calculation using the Survey of Consumer Finances for 1998 and reported in Orzechowski and Sapielli (2003). The latter calculation, done for 1993-2003 using Census Bureau data, yields 0.662. The foreclosure initiation rate comes from Mortgage Bankers Association and is the average number of new foreclosures of conventional fixed rate mortgages initiated each year as a fraction of outstanding conventional fixed rate mortgages for the years 1998-2003 (the data for earlier years are not readily available). The figure is comparable to the one reported in Guler (2008). The ratio of mean housing wealth to mean household networth is from the 1998 SCF and reported in Orzechowski and Sapielli. These authors also report the median household networth for the same survey as $49,932 and the Census Bureau reports the median household income for 1998 as $38,885. The table reports the ratio of these two figures.

To complete the calibration of the model we need to specify the tax schedule $\tau(\cdot)$ and the standard deduction $s$. The tax schedule is chosen to match the tax table for 1998. In our model, people are viewed as individuals (this seems consistent with the earnings data). But we will take individuals to be married. Hence, the tax table we use is the tax table for married filing separately. According to the Census Bureau, male median income of year around full-time workers aged 25 years and older in 1998 was $37,906 and that of females was $27,956. We use the average of these two numbers, which is $32,931, as the median income of an individual filing for taxes. Normalizing the tax brackets for 1998 by this estimate of median income, we obtain the following tax schedule $\tau(\cdot)$:
And, normalizing the standard deduction for a married person filing separately by median income gives $s = 0.1116^2$.

Table 3 displays the steady-state model statistics and the parameter values that generate them. With the exception of the networth-to-income ratio, which is too high in the model relative to the data, the steady state statistics comes reasonably close to their target values.$^3$ The implied preference parameters, $\beta$ and $\gamma$ are not implausible.$^4$ The value of $\lambda$ implies that following a foreclosure a person is allowed back into the mortgage market in 3 1/3 years, on average. The value of $\delta$, which is the depreciation of rental properties is rather small.$^5$

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Target</th>
<th>Model</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>homeowners/total households</td>
<td>0.664</td>
<td>0.645</td>
<td>$\gamma = 2$</td>
</tr>
<tr>
<td>foreclosure initiation/oustanding mortgages</td>
<td>0.0056</td>
<td>0.0049</td>
<td>$\lambda = 0.3$</td>
</tr>
<tr>
<td>mean housing wealth/mean household networth</td>
<td>0.337</td>
<td>0.314</td>
<td>$\delta = 0.002$</td>
</tr>
<tr>
<td>median hh networth/median hh income</td>
<td>1.28</td>
<td>1.48</td>
<td>$\beta = 0.93$</td>
</tr>
</tbody>
</table>

It is useful to discuss some of the more salient features of the steady state. First, one may ask why do people in our model have an incentive to purchase homes? In our calibration, we have assumed that there is no direct benefit to homeownership ($\nu = 0$) and the "rental externality" (the fact that rental properties depreciate but owner-occupied units do not) is negligible ($\delta = \Delta 0.002$). Furthermore, in computing the solution to our model we assume that households who rent can choose their $h$ freely while those who purchase their homes must choose from a finite set of house sizes, i.e., a from a finite set of $k$ (and, therefore, $h$) values. Thus homeownership affords no direct utility benefit, involves somewhat less flexibility (in terms of house

$^2$ Our tax schedule overstates the taxes paid by low-income people because we ignore the earned income tax credit. However, what is important for our study is the tax benefit of owner-occupied housing and this benefit is not affected by the earned income tax credit. This is because the credit is calculated on a person's adjusted gross income and, therefore, does not depend on whether the household rents or owns.

$^3$ A lower $\beta$ would bring networth-to-income ratio closer to the target (people will save less) but it also raises the probability of foreclosure above its target value because the punishment for foreclosure (exclusion from the mortgage market) becomes less effective.

$^4$ The discount factor $\beta$ may seem too low relative to other quantitative macro models. But the possibility of default makes borrowing expensive and to get people to borrow in spite of high interest rates requires the discount factor to be low. Low discount factors also pervade models of unsecured consumer credit as well as models of sovereign debt and default.

$^5$ The value needs to be small in order to make renting sufficiently attractive relative to owning.
sizes) than renting and involves the payment of significant transactions costs in its acquisition. Nevertheless, the majority of people choose to purchase their homes. This is because of the tax benefit of homeownership: the implicit rental income from ownership is not counted as part of income and therefore not taxed. This exemption means that people – especially those in the higher tax brackets – have a strong incentive to purchase their homes. The deductibility of mortgage interest payments merely encourages these households to borrow to finance the purchase of their homes as opposed to self-financing their purchase by accumulating financial assets.

Second, one may ask why are there foreclosures in steady state at all? In steady state the value of a house does not change over time and if the house was purchased with some downpayment the owner can recover his or her home equity by selling the house. So, why default? Two conditions must be satisfied. First, it must be the case that the owner’s home-equity is less than the transactions costs of selling the house – otherwise selling will always dominate default. Formally, the condition is

\[ 1 - \chi S p^* k - \chi (1 + 1/r) < 0 \]  

where \( p^* \) is the steady state price of a unit of housing space. But this condition is not sufficient. It must also be the case that the punishment from default is low. Recall that the punishment for default is exclusion from the mortgage market for several years. This punishment is ineffective for an individual whose current and future income is expected to be low as such a person does not gain from homeownership and would prefer to rent anyway. Therefore, for default to happen the homeowner must have low home-equity and low income.

The third salient feature of our model is that homeowners choose their downpayment and this choice is regulated by the interest rate on the resulting loan. As noted above, a low downpayment is a pre-condition for default and therefore the implicit interest rate on such loans will reflect the probability of default. A borrower can lower the cost of the mortgage by lowering his or her promised payments \( x \) without changing the value of \( k \) - which is equivalent to putting down a higher downpayment since the downpayment is simply \( p^* k - q(k, x, y, w) x \) and the term \( q(k, x, y, w) x \) is generally increasing in \( x \). In steady state, the average ratio of \( q(k, x, y, w) x \) to \( p^* k \) (the loan-to-price ratio) is 0.9127. So, the average downpayment is little under 10 percent. How does this compare to the data? Between 1993-2003, according to the data from the Mortgage Bankers Association, the average loan-to-price ratio on all new mortgages was 0.7748. Note, however, that in actuality households take out subsequent loans against their home equity as well (something which does not happen in our model). Taking these loans into account will likely reduce the discrepancy between model predictions and data.

In relation to endogenous downpayments, it is worth pointing out that our model is consistent with the general view that high income people can borrow at lower costs than low income people. Observe that holding fixed the person’s post-purchase financial wealth, a mortgage for which (8) holds will carry a lower price (higher interest rate) the lower is the person’s earnings \( w \). This is so because for our earningsprocess a person with a lower earnings is more likely to encounter earnings level for which default is the best option. Thus, all else remaining the same, high earning individuals face lower borrowing costs than low earning
A fourth salient feature of our model is that there is positive transactions volume in the housing market. Individuals buy and sell houses. These transactions occur because household earnings change over time and households may wish to either up-size or down-size their housing space as a result. In our steady state, 0.64 percent of homeowners sell each year and 2.06 percent of renters buy each year. The former figure is an order of magnitude lower than the comparable figure in the data. According to data published by the National Association of Realtors, during 1993-2003 the number of single-family home sales as a percentage of owner-occupied units was 4.27 percent, on average. However, our model only contains one motivation for sale – a change in income that makes the existing house size inappropriate (too large or small). In reality, there are other motivations for sale such as relocation and life events (marriage, divorce and change in family size). Thus, it would be surprising if the model came close to matching actual transactions volumes.

Finally, our model is consistent with the observation that owner-occupants consume more housing space, on average, than renters. In our equilibrium the per-capita housing space of renters is 54 percent of per-capita housing space of owner occupants. There are two reasons for this. First, because high earners choose to buy houses the housing space of owner occupants will be greater than that of renters. In addition, the tax benefit of owner occupancy makes owner occupants consume more housing than renters – i.e., if an owner-occupant were to rent he or she would rent less space. This fact will play an important role in generating a mortgage crisis in our model.

7 The Mortgage Crisis

We model the mortgage crisis as stemming from an unexpected increase in the stock of housing. In particular, we assume that the economy (city) enters the initial period, period 0, in the steady-state corresponding to some housing stock $H$. Then, unexpectedly, the housing stock in period 1 rises to $(1+\phi)H$, $\phi > 0$, and stays at that level forever. We assume that the new supply of housing, $\phi H$, shows up as additional endowment in the hands of the financial intermediary sector/business sector. Our analysis will focus on what happens in steady state as as a result of this increase and what happens along the transition to the new steady state.

We conduct our analysis under the assumption that $\phi = 0.05$. This might seem high if our reference point is the whole economy. But by all accounts the housing bubble affected some parts of the country more than others. Thus, the areas that have seen the most foreclosures and the steepest drop in housing prices presumably experienced more overbuilding than the economy as a whole.

Table 4 displays the steady state effects of the permanent increase in the supply of housing. The price of housing space declines by 4 percent and there is about a 2 percentage point increase in the homeownership

\footnote{According to some analysis, long-run demographic and housing trends imply a normal demand for new housing units of about 1.5 million units each year. By this metric, about 2 million “extra” housing units were built during 2001-2006. The average stock of occupied housing units during this period was about 106 million units which would imply an “oversupply” of less than 2 percent.}
rate and about a 1 percentage point decrease in the loan-to-price ratio. There is an increase in the transactions volume (selling and buying) and a decrease in the foreclosure rate.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-shock SS</th>
<th>Post-shock SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per unit of housing space</td>
<td>1</td>
<td>0.96</td>
</tr>
<tr>
<td>Average LTV (%)</td>
<td>91.27</td>
<td>90.31</td>
</tr>
<tr>
<td>% of homeowners</td>
<td>64.55</td>
<td>66.39</td>
</tr>
<tr>
<td>% of homeowners selling</td>
<td>0.64</td>
<td>0.74</td>
</tr>
<tr>
<td>% of homeowners foreclosing</td>
<td>0.49</td>
<td>0.41</td>
</tr>
<tr>
<td>% of renters buying</td>
<td>2.06</td>
<td>2.32</td>
</tr>
</tbody>
</table>

These changes accord with intuition. The (consumption good) price of housing space must decline in order for the economy to absorb the additional supply of housing. A lower price of housing, in turn, encourages more buying and selling because transactions costs (as a percentage of earnings) decline with the decline in the price of housing. To understand why the foreclosure rate and the loan-to-price ratio decline consider, again, the necessary condition for default, namely the condition in (8). Observe that for a mortgage for which this condition holds, a proportionate decline in both $p^*$ and $x$ will cause the difference to become a smaller negative number. In other words, the loss in wealth involved in selling the house as opposed to defaulting on it would be smaller. All else remaining the same this decline in the real cost of selling will lower the probability of default and the cost of the mortgage. Individuals respond to this decline in the cost of borrowing by increasing $x$ (borrowing more) and this accounts for the increase in the loan-to-price ratio. Evidently, however, borrowers (home purchasers) do not increase $x$ to completely neutralize the decrease in the probability of default so that, in equilibrium, there is a small decline in the foreclosure rate as well.

Compared to these long run (steady state) changes in the housing market, the impact effects of the increase in housing supply are dramatically different. Table 5 displays the impact effect along with the old and new steady states.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-shock SS</th>
<th>Impact Effects</th>
<th>Post-shock SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per unit of Housing Space</td>
<td>1</td>
<td>0.89</td>
<td>0.96</td>
</tr>
<tr>
<td>Average Loan-to-Price (%)</td>
<td>91.27</td>
<td>90.67</td>
<td>90.31</td>
</tr>
<tr>
<td>% of Homeowners</td>
<td>64.55</td>
<td>52.01</td>
<td>66.39</td>
</tr>
<tr>
<td>% of Homeowners Selling</td>
<td>0.64</td>
<td>0</td>
<td>0.74</td>
</tr>
<tr>
<td>% of Homeowners Foreclosing</td>
<td>0.49</td>
<td>20.81</td>
<td>0.41</td>
</tr>
<tr>
<td>% of Renters Buying</td>
<td>2.06</td>
<td>1.50</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Observe that on impact (i.e., in period 1) the price of housing declines by 11 percent and the foreclosure rate spikes up to almost 21 percent. Correspondingly, the homeownership rate drops to 52 percent and there is complete cessation of selling activity. Renters continue to purchase homes but the rate of buying on their part drops as well. Indeed, the total measure of individuals participating in the housing market, either as
sellers or as buyers, drops by 37.18 percent. The only statistic that is not very different from its steady state value is the loan-to-price ratio which stays close to 90 percent.

Figure 1 displays what happens to the rate of foreclosures and the price of housing after the initial period. The foreclosure rate falls back to around 4 percent and then rises monotonically toward its new steady state value. The price of housing rises monotonically toward its new steady state value as well. In both cases, the rise is very gradual and convergence takes many years.

What explains these results? In the long run both owner-occupants and renters must increase their consumption of housing space so as to absorb the increased supply of housing. Indeed, in the new steady state the per-capita consumption of housing space of both renters and owner-occupants goes up by 4 percent. But the adjustment by owner-occupants is slowed down by the transactions costs of selling and buying a house. Thus, in the aftermath of the shock, the burden of adjustment falls disproportionately on the rental market and rents decline below their new steady-state value. The decline in rents causes the price of housing space to decline since rents and prices must satisfy (5) with equality: if prices do not fall but rents do, the business sector would have an incentive to sell the property it owns since holding on to it and renting it out would be less remunerative.

The decline in price, in turn, causes the necessary condition for default, condition (8), to hold for a greater number of borrowers. So, the decline in price sets the precondition for foreclosure. But why do borrowers choose to default? In steady state the trigger for default is a fall in income which makes servicing the mortgage payments relatively onerous. But that is not the motivation for default at the start of the crisis. In the period of the shock borrowers default because they wish to take advantage of the low rental price of housing.
space: defaulter give up the tax benets of homeownership in favor of the gain from consuming housing space at a (much) reduced rent. In subsequent periods the trigger for defaults reverts back to bad income shocks.

But exit from homeownership via foreclosure makes achieving equilibrium in the housing market more dif-
ficult. The reason was alluded to at the end of the previous section: all else remaining the same, a renter desires less housing space than an owner-occupant. Thus, as the ranks of owner-occupants shrink there is a tendency for the demand for housing space to shrink as well. Consequently, rents have to fall further to equate demand with supply. Of course, this in turn causes further declines in price which induces more foreclosures. The sharp impact effects on price and foreclosures is the result of this “circular dynamic.”

Figure 2 displays the what happens to transactions volumes. The decline in price makes it very costly to sell a house because the seller has to absorb the capital loss. As a result, there are no sales in the period of the shock. Indeed, there are no sales for several years following the shock. There is also a decline in the percentage of renters buying houses in the period of the shock. This happens because in the period of the shock the ranks of renters is swelled with defaulter and defaulter (or sellers, for that matter) cannot purchase homes rightaway. In the period following the shock there is spike in percentage of renters purchasing homes – this comes from defaulter who are lucky enough to be let back into the housing market. But the percentage of defaulter transiting into homeownership declines over time as the measure of defaulter in the renter pool declines over time. Renters who are not defaulter do not normally wish to purchase homes because they are low earners. These people could be tempted to purchase homes in the expectation of capital gains, but the rise in price is too gradually for that to be profitable.
Why is the rise in price so gradual? Following the period of the shock, the spike in new buyers is roughly offset by new defaulters. Thus, the percentage of homeowners in period 2 is only slightly larger than in period 1. Because the foreclosure rate stays elevated for many years into the future, there is a continuing partial offset to the measure new buyers coming into the housing market. The net result of these dynamics is that the homeownership rate rises back to its new steady state level very gradually (Figure 3). Because homeowners tend to occupy more space than renters (all else the same), the increase in the demand for housing space rises only gradually over time and, therefore, so does the price of housing.

8 The Foreclosure Prevention Policy

This section analyzes the impact of the “loan modification” program implemented by the US government since March 2009. The key features of this policy is as follows. To be eligible for a loan modification the mortgagor’s current income must be less than his income at the time of the origination of the loan (the “hardship” requirement). Furthermore, the ratio of mortgage payments to income must be higher than 31 percent (there are some asset restrictions as well that we ignore). If a mortgage is eligible, the government promises to reimburse investors half the cost of reducing the mortgage payment-to-income ratio from 38 percent to 31 percent. The modification only alters the monthly payments; if the mortgagor were to sell the property, the outstanding loan amount has to be paid. Also, only one modification per mortgage is permitted (i.e., a modified mortgage cannot be modified again) and the modification ends in 6 years after which the mortgage payment is gradually raised back to the original amount. The option to modify an eligible mortgage
is available only for 4 years.

To map this policy into our model we simplify matters somewhat. First, we assume that the percentages 31 and 38 apply to earnings as opposed to income. Since earnings are exogenous in our model this simplification reduces the computational burden significantly. Define $x^m(w) = 0.31w$ and let $x^m(w) = 0.38w$. Then, on a modified mortgage with original payment $x$ and current earnings $w$, the intermediary receives

$$x^m(w, x) = x^m(w) + \frac{1}{2} \{\min\{x, x^m(w)\} - x^m(w)\}$$

and the mortgagor pays $x^m(w)$. The difference is made up by the government. In the event the mortgagor sells the property he must repay $x/(1 + 1/r)$ to the intermediary. Thus, there is no loan forgiveness.\(^7\) We assume that the modification program is offered in the period the shock only (i.e., in period 1 only) and that each agreed-to modification ends in 5 years. Under these rules, the period $t = 1, 2, \ldots, 5$ market value of a mortgage with modified payment $x^m$ and original payment $x$ is given by the recursion

$$q_x(k, x^m, y, w, t) x^m = (1 + r)^{-1} \times E_{w|w}\{$$

$$d_x(k, x^m, y, w', t + 1)[1 - \chi] P(t + 1) k + s_x(k, x^m, y, w', t + 1)[x(1 + 1/r)] +$$

$$(1 - d_x(k, x^m, y, w', t + 1))(1 - s_x(k, x^m, y, w', t + 1)) \times$$

$$[I_{t+1\leq5}(1 + q_x(k, x^m, a_x(k, x^m, y, w', t + 1), w', t + 1))x^m + I_{t+1=6}(x^m + q(k, a(k, x, y, w', t + 1), w', t + 1))x]$$

$$\}

where $d_x(k, x^m, y, w, t)$ is the period $t$ default decision rule of a borrower household with a modified mortgage, given that the unmodified mortgage has payment $x$. Similarly, $s_x()$ and $a_x()$ are the corresponding sale and savings decision rules under modification.

Since $x^m < x$ for an eligible mortgage, the mortgagor will always prefer a modification. But for a modification to actually happen, the lender (intermediary) must not be worse off. Thus an eligible mortgage will be modified if and only if

$$q_x(k, x^m, a_x(k, x^m, a, w, t), w, t) x^m > q(k, a(k, x, a, w, t), w, t) x.$$

Finally, we assume that borrowers currently in the highest earnings state are the only ones ineligible for a loan modification. These people have the same or higher earnings compared to the period in which they took out the loan and therefore fail the hardship test. Borrowers below the top earnings state may fail the hardship test as well but we take these borrowers to be eligible for the program. It turns out that enlarging the scope of the program in this way does not affect the results because the mortgages that do ultimately get modified are almost always the ones that pass the hardship test as well.

Figure 4 displays the foreclosure path with and without the policy in place (in all ensuing figures with the exception of Figure 6, the (red) dotted line corresponds to the path under intervention and the (black) solid line the path under no intervention). The policy is quite effective in reducing foreclosures, but only temporarily so. In the period of the shock the foreclosure rate rises to little over 8 percent, which is less

\(^7\)The policy permits forgiveness of interest but this is an option not a requirement for modification.
than half of the spike in foreclosures in the absence of the policy. The foreclosure rate remains below the non-interventionist rate for as long as the modifications remain effective (year 6 in the Figure).

Once the modifications expire, there is a second spike in the foreclosure rate of more than 9 percent and the foreclosure and the foreclosure rate remains higher than non-interventionist path for quite some time. The rates with and without intervention converge around 25 years in the future.

Why explains this pattern? As noted earlier, the main motivation for foreclosures in the period of the shock is to take advantage of the low price of rental housing space. This incentive is blunted if the homeowner can reduce his or her mortgage payment – which is exactly what the loan modification program does. Consequently, as long as the homeowner can continue on in the house with lowered payments and does not receive an adverse income shock, he or she does not wish to default. This lowers the foreclosure rate for long as the modification is in place. Once the modification expires, the incentive to take advantage of the low price of housing space re-emerges and the foreclosure rate spikes up.

Figure 5 displays the path of prices under the two scenarios. There is less of a decline in prices in the period of the shock and prices actually recover some during periods 2 − 6. Then there is a crash in period 7. Following the second crash, there is a gradual recovery in prices.
The path prices basically mirrors the homeownership rate under intervention. This is evident from the comparison in Figure 6. The homeownership rate rises between periods $2 - 6$ and then declines, and, correspondingly, the prices rise and then decline.

Figure 6
In this discussion, we have simplified matters by assuming that the modifications expire at the end of 6 years. In reality, the plan is for the modifications to be taken away gradually. The gradualness may make the second rise in foreclosures less sharp but it is unlikely to completely eliminate it. The temporary nature of foreclosure relief suggests looking at the effects of policy from a cumulative perspective. Are there fewer total foreclosures as a result of the intervention? Figure 7 shows cumulative path of foreclosures.

![Figure 7](image)

Evidently, the policy reduces total foreclosures for about 20 years out but then leads to slightly higher total foreclosures in the very long run.

9 Conclusions

(To be added)

References


