Interest Accumulation in Retirement Home Equity Products

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This version: September 3, 2010

Abstract

Reverse mortgages and similar products generally feature negative amortization: no principal or interest payments are made to the lender until the borrower moves out of the home or dies, so the balance due grows with time. This paper shows that when markets are otherwise complete, replacing negative amortization with interest-only payments and mandatory purchase of life annuities sufficient to pay interest on withdrawals increases lender profits and borrower welfare. Rationalizing negative amortization requires special combinations of forms of market incompleteness, among which may be private information on mortality and exogenous price variation.

1 Introduction

The large share of retiree wealth in owner housing, combined with a common desire to "age in place," suggests that reverse mortgages or other home equity loans to retirees would offer considerable welfare benefits.[1] There are two potential sources of gains to a transfer of funds

*I thank Joe Kelly of New View Advisors for helpful discussions about the US reverse mortgage market. Seminar participants at the University of British Columbia, University of Maryland, and the NBER Aging Group provided helpful comments.

[1]The theoretical gains are demonstrated by Artle and Varaiya (1978). Mayer and Simons (1994) argues that large numbers of elderly homeowners would reap considerable gains from taking on reverse mortgage debt. Tests of non-annuitized income and assets may limit conventional mortgages and other credit available to older homeowners.

from a financial institution to a homeowner that is reversed when the homeowner sells the home or dies. First, if bequest motives are weak and the loan terminates with death, then the implicit transfer from the borrower's estate to the borrower makes the borrower better off. Second, if home equity is eventually sold late in life, home equity loans transfer resources from the cash-rich period after sale to the cash-poor period before. If utility is concave over expenditures, and the marginal utility of expenditures is no greater after moving than before, this may also increase borrower welfare.

In practice, reverse mortgages and similar products in the US and elsewhere have not proven popular, and retired homeowners throughout the developed world owe very little mortgage debt of any form.[2] The gap between the theoretical benefits and revealed demand for retirement home equity products invites the question of whether product design is close to optimal. Home equity product design is a difficult because profits depend on home prices that are volatile and borrower's stochastic mortality, and borrowers' stochastic, endogenous, and complex choice of whether and when to sell the home if alive.

This paper investigates the role of a pervasive but seemingly peculiar feature of common home equity products for seniors: negative amortization. When all markets are complete other than that for home equity, the value of the home is constant over time, and consumer utility satisfies a natural baseline set of assumptions, negative amortization is not a desirable contract feature. Under those assumptions, a Pareto optimal contract provides for 100% loan-to-value and interest only payments as long as the loan is in effect. Interest in the optimal contract is paid out of a mandatory life annuity purchase that provides constant payments whether the borrower remains in the initial home or not. This "annuitized reserve" contract has the virtue of leaving the lender indifferent to the date at which the borrower chooses to move. The same is generally not true of a negative amortization contract. A mortgage that

[2]In the US, less than one percent of retired homeowners owe reverse mortgage debt, and approximately twelve percent of fully retired households in the 2004 wave of the HRS/AHEAD survey owed any mortgage debt at all (Davidoff (forthcoming)) In the UK, an industry group (of Mortgage Lenders (2009)), reports that member financial institutions originate roughly 30,000 loans per year, suggesting a somewhat larger share of eligible homeowners (from of Mortgage Lenders (2009)).

forces the borrower to commit at origination to a fixed date of exit from the home would presumably not be desirable to real-world borrowers who learn about preferences over time.[3]

If the value of the borrower's home is stochastic or a function of borrower behavior, and the mortgage terms cannot be conditioned on ex-post realizations of prices, then the "annuitized reserve" contract may not achieve a Pareto-optimal outcome.[4] In those cases, lender profits may depend on the date of termination (and possibly borrower investment behavior). With markets thus incomplete, it is difficult to characterize an optimal contract and to know if the simple annuitized reserve contract will Pareto dominate a negative amortization contract.

To evaluate the negative amortization feature under the plausible conditions that prices are time-varying and subject to investment I model a retired consumer who owns a home, faces a time-varying utility cost of moving, and can only extract home equity by moving or with a single take-it-or-leave-it reverse mortgage offer from a single lender. I ask if the lender can achieve greater profits while offering a fixed level of expected utility to the consumer with a negative amortization or annuitized reserves contract. I briefly describe why the annuitized reserve contract may be Pareto optimal, and compare this product to existing home equity products in Section 1.1. I describe the model and parameter ranges in Section 2. Section 3 summarizes measures of the welfare benefits to borrowers and lenders that arise from replacing negative amortization with annuitized reserves in different environments. Section 4 concludes.

To highlight the role of negative amortization, I offer numerical examples that restrict the available gains from annuitization of principal. Negative amortization and annuitized reserves offer different levels of partial annuitization. Whether due to these or other "rational" considerations or to some failure to optimize, demand for annuitization of private savings is

[3] I address the sources of changing preferences indirectly, as discussed below.

[4] Chinloy and Megbolugbe (1994) and Miceli and Sirmans (1994)evaluate the profitability of reverse mortgage contracts such as HECM in these cases. Shiller and Weiss (2000) highlight the possibility of maintenance moral hazard. The novelty of this paper is to consider the relative merits of negative amortization versus annuitized reserves in these settings.

weaker than many models predict.[5] Further, reverse mortgage proceeds can be annuitized outside of the reverse mortgage. I allow for such annuitization and for a moderately strong bequest motive. Whether bundling pure home equity loans with life annuities for the principal amount would increase demand for both or either component is an interesting question that is beyond the scope of this paper.[6]

1.1 Amortization and Home Equity Product Design

Consider a homeowner uncertain when she will die who derives utility from living in her home and spending money, but is only able to spend home equity by selling and exiting her home, or by tapping a single home equity product. Suppose first that this homeowner:

Assumption 1. *1. faces no uncertainty other than date of death;*

2. does not have private information about her mortality;

3. discounts at the same time-invariant rate r as everyone else in the economy (the lender);

4. does not derive utility from wealth left to heirs;

5. can not affect the value of the home, e.g. through maintenance;

6. for any x, derives equal utility from (a) living in the home and consuming $x - rp$, where p is the time-invariant value of the home, or (b) rents a different home and spends x gross of rent.

Under these 6 assumptions, no contract Pareto dominates a bundled fair constant real annuity financed by a non-recourse 100% loan-to-value forward interest-only mortgage (equiv-

[5] see e.g. Chalmers and Reuter (2009) for a critical survey of the empirical literature.

[6] A reasonable intuition is that negative amortization offers greater annuitization of capital gains than annuitized reserves when prices are stochastic or subject to investment. This is not true if an annuitized reserve loan can be made for greater than 100% loan-to-value and at an interest rate greater than the risk-free rate.

alently a sale-leaseback) that the borrower terminates at her discretion.[7] This contract is Pareto efficient because it eliminates the only relevant forms of market incompleteness under autarky: (a) no market for unintended bequests and (b) inability to spend home equity without leaving the home. Under this contract, if the mortgage interest is equal to the required rate of return on the home's value, the lender is indifferent to the date of loan termination. The borrower is thus free to choose the optimal date of a move (if any while alive) to rental housing. The constant real annuity provides the same constant consumption stream that the retiree would choose if faced with a complete set of assets (provided at zero profit by the lender) paying only contingent on survival.

The assumption of no bequest motive may not be relevant to the results below concerning the treatment of interest on outstanding principal. As shown by Yaari (1965), as long as fairly priced annuities are available and the bequest motive is utilitarian, principal is optimally split between an annuity and an immediate gift. A bequest motive (or other demand for liquid savings, such as incomplete markets) would matter if annuities are not fairly priced.

The most common home equity loans geared to seniors (CHIP in Canada, HECM in the US, and Drawdown Roll-Up mortgages in the UK) work approximately as follows:

- Borrowers must be above a threshold age (60 in Canada, 62 in the US, 50 in the UK);
- At origination, a line of credit is extended to the borrower;[8] some fees and costs are financed as an initial draw;
- The borrower uses the line of credit as desired up to a time varying limit on the outstanding balance;
- The borrower remains responsible for property taxes, maintenance, and insurance;

[7] Such a bundled annuity and forward mortgage is available in the UK (the "Home Income Plan"), but not the US or Canada, and is not the most commonly sold product in the UK.

[8] Simulations reveal that a line of credit Pareto dominates a required, non-prepayable lump sum advance. Given multiple sources of market incompleteness, this result is not guaranteed, but from the consumer's perspective, conditional on design, the lump sum requirement is a weakly costly constraint. Intuitively, the lump sum should offer greater annuitization at the cost of a less favored consumption stream and worsened moral hazard on termination.

- Interest on the outstanding credit balance is periodically added to the principal due;
- (Negative amortization) The maximum outstanding balance grows at approximately the stated interest rate on the loan, inclusive of guarantee and servicing fees;
- The loan terminates by sale of the property, death of the borrower (both if a couple), or (less commonly) through voluntary prepayment;
- The lender has a claim senior to the borrower on sale proceeds up to the accumulated principal and interest;
- If equity remains on loan termination above principal and interest due, the borrower or their estate gets this excess;
- The lender has no recourse to other borrower assets if the outstanding balance exceeds the value of the home at termination.
- The "lender" may be an individual company or, as in the US, a mixture of the US government (providing guarantees), intermediaries, and owners of mortgage backed securities.

Two features of this family of contracts are notably different from the hypothetically Pareto optimal contract described above. First, annuitization (insurance against mortality) of principal is incomplete unless (a) the whole loan balance is drawn immediately, (b) there are no moves while alive, and (c) the interest rate on the loan is infinite.

Second, if the accumulated principal and interest exceed the value of the home, then other than property taxes and maintenance, there is no capital cost to the borrower to remaining in the home a bit longer (and there may be a negative cost if prices are volatile). Presumably, rental housing would include property tax payments, maintenance costs, and an opportunity cost of the landlord's capital. Thus the borrower perceives an artificially low cost to remaining in the home. Alternatively, if the borrower terminates the loan while

equity remains, if the interest rate on the loan exceeds the ratio of rent to price (here assumed to be r, then an incremental period in the home will cost more than rental housing. These discrepancies between the borrowers cost and the true capital cost of the home will generally induce borrowers to move at a date different from the Pareto optimal move date that would be agreed on if that date could be contracted.

Thus the negative amortization loans described above area equivalent to the "Pareto optimal" only contract only under the unrealistic conditions (a) through (c) above. However, the conditions in Assumption 1 that guarantee Pareto optimality of the annuitized loan are themselves unrealistic. Prices are stochastic, and shocks to health and events in the lives of family and friends affect the marginal utility of expenditures, and the difference in utility associated with moving out of the home. Once prices are time-varying, the Pareto optimal contract becomes more difficult to implement, because the date at which the borrower moves will affect the present value of cash flows between the borrower and lender. Even if prices are deterministic, changes to the value of interest payments may render the marginal utility of consumption different before and after moving, distorting the choice away from first-best under a contractible move date.

To address these complications and to get an idea of the robustness of the complete markets result that negative amortization is undesirable, I describe a somewhat more general evaluation framework in the next section.

2 Evaluation Framework

I assume that borrowers have access to fair immediate life annuities outside of the available loans at both loan origination (date 0) and any date of a move T. If no equity remains at T and there is no uncertainty, markets are complete. I do not allow the consumer to purchase a constant real annuity that pays only between dates 0 and T, although there might be demand for such a product. I assume in this version that the cost of a jump in consumption

at the date of sale by the annuitized remaining equity, if any remains, is small. For this reason, I assume that the balance of either loan is drawn immediately and divided between a bequest and a life annuity.[9]

The profits on a negatively amortizing loan of size x are given by:

$$\pi_n = -x + \sum_{t=0}^{T} q_t m_t \min\left(x\left[1+r_m\right]^t, p_t\right)\left[1+r\right]^{-t} + q_T \min\left(x\left[1+r_m\right]^T, p_T\right)\left[1+r\right]^{-T}. \quad (1)$$

In equation (1), q_t is the probability of survival to date t as of date 0. m_t is the probability of death at t conditional on survival to t.

Profits on an annuitized reserves loan, assuming that the annuity component is fairly priced, are given by:

$$\pi_a = -x + \sum_{t=0}^{T} q_t \left[m_t \min\left(p_t, x\right) + r_m x\right] + q_T \min\left(x, p_T\right)\left[1+r\right]^{-T}. \quad (2)$$

The annuity component consists of reserves equal to

$$z = \sum_{t=0}^{\infty} q(t) x r_m [1+\hat{r}]^{-t}, \quad (3)$$

and life annuity payments in the same expected amount if $r_m = r$. These annuity payments are positive income to the lender up to date T, because they are paid to the consumer, who has paid this in reserves to the lender and also owes this amount as interest on the constant balance x.

With complete markets, the discount factor in annuity reserves $r_m = \hat{r} = r$. With stochastic prices and the possibility that the mortgage balance x might exceed p_t, the mortgage interest rate for zero profits must be greater than r. In that case, the suitable discount rate

[9]Reverse mortgages featuring regular monthly payments would seem to generate less mobility moral hazard, as they become "underwater" later, but these have not proven popular, at least in the US.

for the annuity is not clear, and I consider a range between r and r_m in the simulations below. In principal, the reserves could be greater than the loan amount, but they are not in any parameter cases I choose.

Bequest motives, contracting problems, and realistic uncertainties may reduce (or increase) the value of annuitizing wealth, with the direction of the effect on demand complicated and dependent on strong assumptions.[10] To isolate the effect of negative amortization in a setting with some claim to generality, I only consider annuitization of interest reserves and do not allow loan proceeds advanced to borrowers to be annuitized. Because loan balances and interest rates will differ between negative amortization and interest reserved accounts, it is not clear which contract will generate larger transfers from the short-lived to the long-lived.

Inspection of equations (1) and (2) show that the lender in a negative amortization contract is exposed to a greater risk of "crossover," such that the value of the home at termination is less than the balance due. This involves a transfer from the lender to the borrower in states of the world in which prices are low that must be matched by a similar transfer in states of the world in which prices are high.

I consider a consumer with expected lifetime utility given by

$$\begin{aligned} U = & \sum_{\omega} f_{\omega} \sum_{t=0}^{T} q_{it}[1+r]^{-t} \left[\frac{c_t^{1-\gamma} + y}{1-\gamma} + bw_t \right] + \\ & \sum_{\omega} f_{\omega} \sum_{t=T+1}^{A} q_{it}[1+r]^{-t} \left[\frac{[c_t - rp_0 + y]^{1-\gamma}}{1-\gamma} + bw_t + k_t \right] + \alpha k_T q_{iT}[1+r]^{-t}. \end{aligned} \tag{4}$$

Wealth w_t is equal to unannuitized savings transferred to heirs at time t. With the availability of fair annuities, wealth will only be held and transferred at dates 0 and t. I assume without proof (and as yet without numerical verification to save computation) that consumers (considered below) facing unfairly priced annuities behave in the same way. Such consumers might wish to finance some consumption with cash. f_{ω} is the probability that

[10] See Davidoff et al. (2005) for a discussion.

state of the world ω arises.

In equation (4), q_{it} is the probability that the retiree lives to period t as of period 0. r is the common discount and interest rate. γ measures the curvature of utility in consumption.[11] Utility differs from that given in standard life cycle models by the presence of costs of selling the home at date T. At T, the consumer faces immediate disutility (or possibly utility) αk_T. α measures the relative strength of the immediate versus ongoing disutility. At all dates thereafter, the consumer must spend rp_0 on rent and faces additive utility or disutility k_t. y denotes a pension of varying size.

Given the relative novelty of the specification, some remarks are in order. The setup is different from that posed by Artle and Varaiya (1978) in two ways. First, mortality is stochastic. Second, the difference between remaining in the home and moving is not the inefficiency of living in rental housing (rp_0 is the exact financial opportunity cost of remaining in the home), but instead the additive utility or disutility of being out of the original home. I make the natural assumption that k is increasing in t.

An important question for product design is whether the marginal utility of expenditures should be greater before or after leaving the home. With a non-distortive and 100% loan-to-value financial product or a sale leaseback, there is no difference in the specification above. Flexibility in choice of housing after moving suggests greater marginal utility after moving (see, e.g. Chetty and Szeidl (2007)). The declining pattern of k and z is motivated by declining health. Moves out of owner housing are frequently to assisted living facilities or to the homes of relatives. In the former, and particularly the latter, demand for real estate may be lower than it would be in housing for the healthy. Thus the pure effect of moving (as opposed to aging and its attendant increasing frequency of medical problems) on marginal utility of expenditures is thus ambiguous. With no convincing estimates available, a zero effect seems like the most natural choice.

I assume that the retiree has access outside of the reverse mortgage to immediate constant

[11] "Risk aversion" is difficult to characterize given the option of timing the move.

real annuities. Under some circumstances, an annuity that distinguishes between the period before T and after T would generate an improvement over a constant real annuity. Given the generally late date of moves, I assume that this market incompleteness has a small effect on welfare and a smaller effect on the difference in maximal welfare between the negative amortization and annuitized reserves mortgages.[12]

To further address the possibility that demand for annuities, rather than move date distortions, drive the results, I consider variation in the linear bequest b. This literally represents a desire to leave wealth to heirs, but bequests and precautionary savings are difficult to separate behaviorally and this formulation should provide some generality to cases where there is a demand for liquid savings other than through a bequest.

Given savings only in liquid annuities, letting I_a denote an indicator for an annuitized reserve product, non-pension consumption at date t is given by:

$$c_t = \frac{x - w_0}{\sum_{\tau=0}^{\infty} q_\tau [1+r]^{-\tau}} \qquad t <= T \qquad (5)$$

$$c_t = \frac{x - w_0}{\sum_{\tau=0}^{\infty} q_\tau [1+r]^{-\tau}} + \frac{\max\left(p_T - x\left[1 + r_m\left[1 - I_a\right]\right]^T, 0\right) - w_T}{\sum_{\tau=t+1}^{\infty} q_\tau [1+r]^{T-\tau}} + I_a r_m x \qquad t > T \qquad (6)$$

When only mortality is stochastic (through fixed q) the consumer's problem consists in choosing T, w_0 and w_T conditional on an offer from the lender. In addition to survival, I allow prices to be either stochastic or a deterministic function of borrower behavior. I fix the initial value of the home at $p_0 = 100$. I consider two simple forms of price uncertainty. In the first, immediately after executing the mortgage, the borrower learns whether prices will fall by 1% or rise 1% per year every year until death. In the second, prices jump by 20% or fall by 20% immediately after loan execution. Neither of these are accurate representations of the evolution of prices, but they provide considerable price variance, and avoid the need

[12] With incomplete information, short horizon annuities play a meaningful role, and their existence will affect the returns on life annuities.

to take a stand on the complex comparative statics effects of price variance on the optimal move date and on savings and consumption choices. I also consider each of these price paths in a situation where the determination of price growth is made by the borrower rather than fate. In some specifications, the borrower may spend 3 units of consumption in period 0 in exchange for having getting more rather than less appreciation (she obtains 3 extra units of wealth in period 0 if she chooses less appreciation.

I consider the two products described above and vary the parameters described in Table 1. For each economic environment, I estimate welfare as follows:

- Fix an economic environment, a combination of: bequest strength b, pension level y, constant $p_t = 100$ or the price paths and probabilities as discussed above (with increases or decreases equally likely), the ability of the consumer to determine prices through investment, limitations on the loans (maximum loan $x = 100$ and maximum interest rate (4.5% or 6.5% relative to $r = 2.5\%$.

- The lender offers all combinations $\{r_m, \hat{r}, x, \}$ within some range in negative amortization and annuitized reserve form.

- For each offer, the borrower maximizes expected utility given the best choice of bequest gifts, annuity purchases, investment in the home if applicable, and move date while alive T, if any.

- Calculate profits given the environment loan terms, price trajectory, and termination date, assuming q_i is the female survival probability described in Table 1;

- Choose as a baseline utility $\bar{u}$ the maximal utility across all annuitized reserve offerings for a given environment.

- Calculate the maximum feasible profits that the lender can earn in a negatively amortizing loan where an optimizing consumer attains utility no less than $\bar{u}$. I call this the "compensation" associated with the environment, something akin to a compensating

or equivalent variation measuring the willingness of the lender to pay for the ability to issue an annuitized reserve rather than negatively amortizing mortgage.

- Calculate the difference in differences between (1) [(a) the valuation of the selected interest reserve loan and the selected negatively amortizing loan] for a consumer with male rather than female survival probabilities (less longevity) and (2) [(a) - (b)] for the original female consumer on whose longevity annuity prices and profits are calculated. If the difference (1) - (2) is negative, then people likely to die older may be viewed as likely to select into the annuitized reserve loan. By construction, (1)-(2) $\approx$ (1).

2.1 Parameterization

See Table 1. Many of the parameters have not, to my knowledge, been estimated explicitly. A curvature parameter $\gamma = 2$ is a moderate degree of risk aversion. Bequest strength is a matter of some controversy and is difficult to separate from demand for precautionary saving (see, e.g. Skinner (1996).) I thus consider both retirees with no bequest motive and those with parameter $b = .4$, for whom the marginal utility of expenditures equals the marginal utility of a bequest at $1/c^2 = .4$ or $c = 1.6$.

Empirically, a large fraction of homeowners die without moving, and moves while alive are often triggered by incapacity or death of a spouse.[13] To model exogenous moves without stochastic taste shocks, one can simply impose a large penalty to moving at a different time from some specified optimum. I specify the disutility of moving at a particular date as the difference in utility from consuming between the 10th and 90th percentiles of the distribution of social security income among recipients in the 2000 PUMS sample (from IPUMS). This value changes with age, as I multiply the difference in utilities by the difference between the date t and age 90. I choose 90 as an inflection point in the cost of moving because ownership begins to decline sharply around age 90. I consider date 0 to be at age 75. While eligibility for home equity extraction is generally earlier than 75, there is no reason to tap a home

[13] See, e.g. Venti and Wise (2000) or Megbolugbe et al. (1997).

Table 1: Parameterization

Variable	Symbol	Values	Rationalization
Utility curvature	u	2	Moderate in empirical estimates
Bequest weight	b	0 and .4	Wide range
Survival	q_t	social security	Consider female (high) and male (low)
Discount/interest rate	r	2.5%	Historically moderate 10 year TIPS
Out of home disutility	k_t	$[.1, 1.0] \times [u(18) - u(4)]$*(age-90)	90-10 percentiles of social security income in US census ownership drops rapidly +/- 90
Initial age	$t = 0$	75	
Date of move multiplier	α	3	Introspection
Price of home at $t = 0$	p_0	100	Moderate for low wealth households
Pension	x	0 to 3	Low social security, net of some costs
Appreciation	p_t	+/-1%/year or +/- 20% 1-shot	Wide range
Rent after T	rp=3	Marginal utility equalization	

equity loan before exhausting other more liquid savings. Empirically HECM borrowers are older than the eligible population.[14] When the disutility multiplier is small, the move is more endogenous to housing wealth and costs, when larger, more exogenously given. A natural intuition is that more endogeneity will make negative amortization relatively worse.

As discussed above, when prices are constant, I equalize marginal utility of equal expenditures before and after moving when mortgage payments are equal to rp_0, the rent of fixed housing consumption. Equal marginal utility strikes me as the most reasonable assumption given competing considerations such as health needs, flexibility after moving, and the possible over- or under-consumption of location, square footage, and yard space before moving.

I use the male and female survival probabilities from current US Social Security life tables. I truncate the age distribution by assuming death for sure conditional on survival to age 110.

3 Results

A first remark on results is that the problem is generally not globally concave in the move date T. Even with the annuitized reserve product and deterministic prices, Figure 1 shows

[14] Rodda et al. (2000).

that maximized utility conditional on choice of move date T is concave in T up to 15 (age 90 starting from 75) and convex thereafter. The convexity comes from decreasing mobility costs as the and decreasing survival probability as T increases.[15] In some cases (when there is a bequest motive and low mobility cost and a negative amortization mortgage) the corner solution of never moving may be optimal.

The numerical solutions to the problem may be characterized as follows: in almost all economic environments (184 out of 192 parameter combinations), profits must be negative for a negative amortization loan to provide equal utility to an annuitized reserve loan. Selection, though, is likelier to be adverse with the annuitized reserve (the difference in differences in valuation is negative in 166 of 192 cases).[16]

Both of these main results conform with intuition. When the price of the home is constant and markets are complete, the structure of preferences guarantees that the annuitized reserves loan will be preferred. As discussed below, the variability of housing prices introduces multiple considerations that affect the relative desirability of negative amortization in both directions. As for selection, consider a consumer who knows that she will die very soon. Because health here is deterministic, an interior move T will not occur, so cash up front is relatively more important than reduced principal obligation later. Thus for individuals with lower (male) survival probabilities, the negative amortization loan is likely to be relatively more appealing.

The magnitude of the compensation lenders require to be indifferent to offering a woman with average survival prospects a negative amortization loan instead of an annuitized reserve loan ("compensation") is generally small, but quite variable. Across all simulations, the median required compensation is 1.4 (thousand), the mean is 2.2, the maximum is 37.17, and the minimum is -6. The differences across environments are driven primarily by the

[15] This non-concavity justifies a numerical approach, as the lender's problem would include a constraint on borrower's optimizing her move date that even in continuous time could not be captured with only the first order condition $\frac{\partial U}{\partial T} = 0$.

[16] A better measure, left to future work, might be the difference in equivalent variation, but even this would be imperfect.

considerations described below.

3.1 Bequest motive

With or without price volatility, the presence of a linear bequest motive dramatically reduces the cost of negative amortization. With no volatility, the mean product difference is 3.1 when there is no bequest motive and .69 with a bequest motive. With price volatility, the means without and with a bequest motive are -4.4 and -.2.[17]

The key mechanism is that when utility is less concave in wealth, the borrower is more willing to take wealth in high price states of the world (by leaving immediately) and low states of the world (by never moving) in exchange for moving away from the internally optimal date. With a bequest motive and, the interquartile range of moves (across other environment variables) when price appreciation is low or zero in the negative amortization product is 16 to 17.25 versus 15.75 to 15.75 with annuitized reserves loan. With no bequest motive, that range is 16 to 17.25. In the case of price appreciation, the interquartile range with an without a bequest motive are 11 to 35 versus 0 to 35 with a bequest motive.

3.2 Exogenous versus endogenous appreciation

The maximum compensation required to be willing to use negative amortization of 37.2 occurs when there is no bequest motive, prices are volatile, and owner investment determines appreciation. With no bequest motive, the interest rate on a negatively amortizing loan is sufficient that the appreciation gain is insufficient to leave equity at the disutility-minimizing move date of 15. Thus there is little incentive to spend the required 3 units of consumption in period 0. With an annuitized reserve loan, the gain to investment is realized at the optimal move date because the principal balance is not rising over time. Thus the profitable investment is undertaken in an annuitized reserve loan, but not a negatively amortizing loan,

[17] This difference does not rely on the linearity of the bequest motive, and indeed is stronger in a parameterization with concave bequest utility, where the marginal utility of bequeathed wealth is not a function of exogenous housing price changes.

creating a large gap in valuation.

The minimum compensation required (-6.0) occurs when appreciation is exogenous, there is very low cost of moves different from age 90, the retiree has a pension earning 3 per year, and there is a bequest motive. The reason is this: negative amortization loans require an interest rate spread reflecting the moral hazard on the move date. This moral hazard arises because the "underwater" mortgagor has no incentive to leave the home, and thus stays beyond the disutility-minimizing age, a Pareto inferior outcome that would lead to negative profits if $r_m = r$. This interest rate spread has the virtue of transferring resources to borrowers who and their estates who live a long time and in the low appreciation state of the world to borrowers who die young and in the low appreciation state. As it turns out, the optimal annuitized reserve loan never involves a rate spread, but this is only because the move distortion outweighs the gain from redistribution. In the presence of a bequest motive, the market incompleteness associated with an assumed inability to hedge price risk renders the negative amortization contract superior to a second best. Notably, the gain to negative amortization in this highly incomplete market setting is reduced by approximately 40% when "usury" is banned, and the maximum rate allowed falls from 6.5% to 4./5%. With complete information, the only potential benefit of negative amortization is the smoothing of consumption across price appreciation states of the world. As the interest rate spread goes to zero, this smoothing is reduced.

In this highly favorable case for negative amortization, there is an immediate move if prices jump at time zero, and no move while alive if they fall at date zero. The behavior induced by the combination of low disutility of moving and linear bequest does not match reality in terms of the timing of voluntary exit through sale. In this case, with exogenous appreciation, the borrower moves immediately if prices fall 20 percent immediately, and never moves if prices rise. As shown by Davidoff and Welke (2006), reverse mortgage terminations have instead been procyclical. In the simulations, procyclical mobility is more likely to arise when maintenance is endogenous and when there is no bequest motive, conditions less

favorable to negative amortization.

4 Conclusion

Negative amortization provides an illusion of mortgage "affordability" because borrowers do not need to make payments while alive and in their home. However, the lender's cost of capital must be paid through some combination of an interest rate spread due at loan termination or reduced loan-to-value. Thus across the simulated environments I consider, the mean difference between the loan proceeds available in negative amortization and annuitized reserves loans is just 12.5% of the difference between 100% and the annuitized reserve loan proceeds. By contrast, the interest rate on the negative amortization loans has a mean spread of 70% of the riskless rate, and the optimal annuitized reserve loan is always at the riskless rate, with risk dealt with through smaller loan to value.

When mortality probabilities are common information, in almost all environments I consider, lenders would require compensation to replace the less distortive annuitized reserve loan with a negative amortization loan along the lines of what is commonly seen in markets. The significant costs of negative amortization are a distortion of the timing of when to move while alive and a related reduction in the demand for investment in projects that provide high price appreciation with little dividend benefit.

Beyond an appeal to some kind of consumer irrationality, there appear to be two sets of justifications for negative amortization. The first justification is that the trade of larger up-front proceeds for less income or wealth after moving is more palatable to consumers with low expected survival. Such customers may provide greater profits to lenders, depending on certain parameters. A famous case of longevity driving profits very negative was Mmme Calment of France, who remained alive for roughly 50 years past signing a one-to-one home equity contract. Adverse selection does not seem to be an important motivating factor in the US: loans are priced based on the age of the youngest borrower, with no actuarial adjustment

for the sex or marital status of that borrower. Generally, one would expect retirement home equity products to appeal to lower human capital consumers with shorter longevity than population averages.

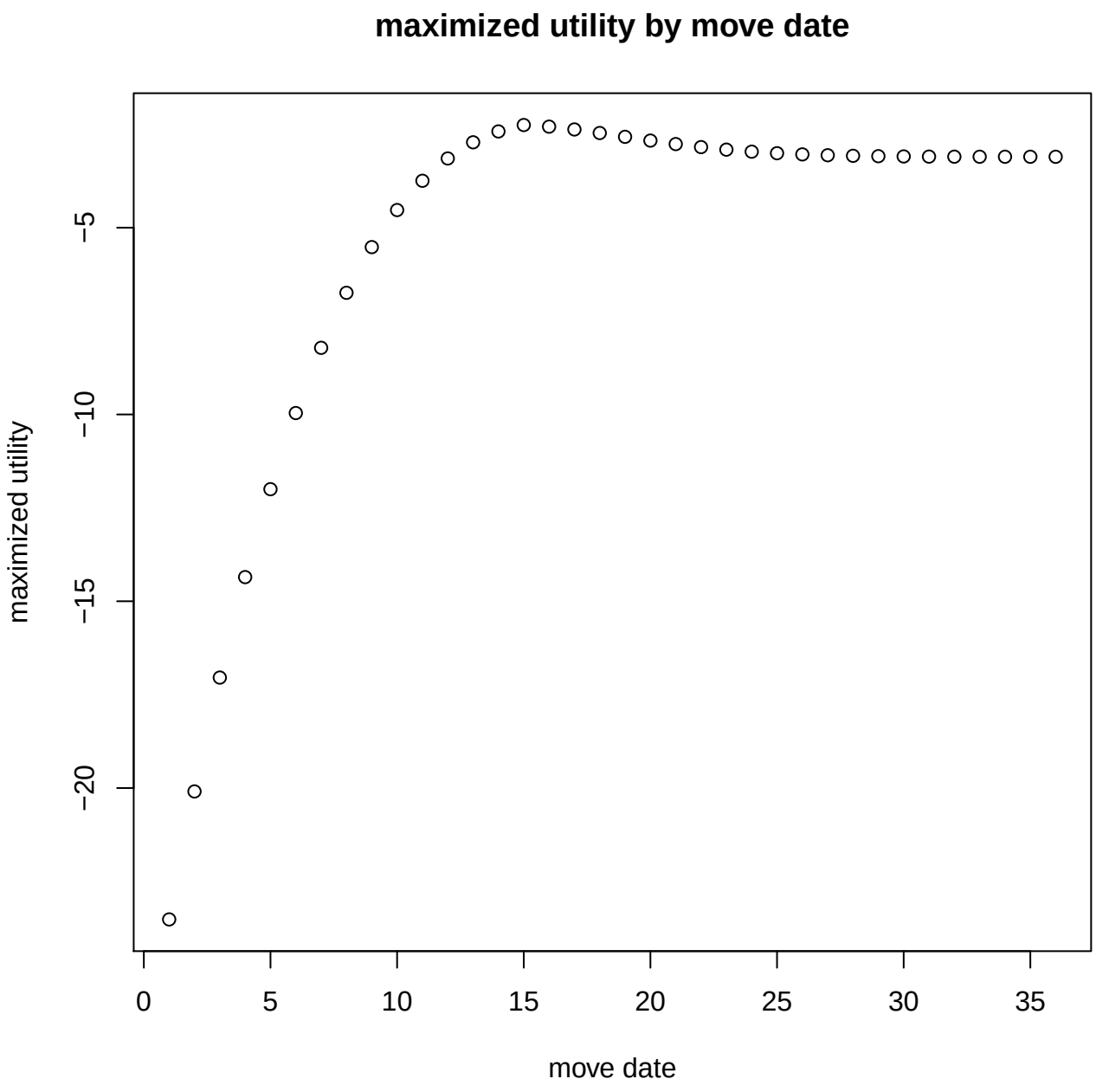
A second justification for negative amortization is the simultaneous presence of large exogenous price volatility, highly incentive-sensitive exit from the home, and a lack of demand for consumption smoothing across states of nature, perhaps driven by a linear bequest. Exogenous price volatility will surely be on the mind of homeowners throughout the developed world in coming years. Whether these considerations are salient to marginal retired home equity consumers brings up several interesting and open questions. Can and should reverse mortgage contracts credibly condition on an aggregated price index? Are marginal home equity borrowers interested in consumption smoothing? How willing are they to trade off the unpleasantness of moving at the wrong time against financial incentives? Generally, these conditions would be unfavorable for demand for any home equity extraction product, and given small market shares, one might expect the marginal consumer to have a high underlying predisposition for a well-modeled product.

As noted above, US reverse mortgage borrowers appear more sensitive to price fluctuations in their mobility choices than older homeowners generally, and in a way that suggests the costs to negative amortization would be large. This paper models demand for moves while alive in a highly ad-hoc way. Realistic integration of health and financial risk applicable to some kind of representative household's endogenous mobility choice may not be feasible, but progress in that direction will have important consequences for home equity product design.

I have assumed throughout that 100% of loan proceeds are issued at origination. This may heighten the moral hazard on loan termination, and a preferable contract might involve a line of credit. In the US and elsewhere both a line of credit and lump sum are available. The line of credit might generate a later "crossover" date if credit use is slow. In this paper, I have assumed that consumers wish to annuitize or gift all proceeds, so a line of credit

would not slow the growth of the mortgage balance or soften termination moral hazard. For a consumer with limited bequest strength but facing unfair annuity prices and high demand for precautionary savings, matters might be different.

Figure 1: Maximized utility conditional on choice of move date T for a consumer facing a 100% loan to value annuitized reserve mortgage with interest rate r_m equal to discount and riskless rate r.



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