Run-up in the House Price-Rent Ratio: How Much Can Be Explained by Fundamentals?*†

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Abstract

This paper studies the joint dynamics of real house prices and rents over the past decade. We build a dynamic general equilibrium stochastic life cycle model of housing tenure choice with fully specified markets for homeownership and rental properties, and endogenous house prices and rents. Houses are modeled as indivisible, discrete-size durables which provide shelter. Homeownership confers access to collateralized borrowing, provides sizeable tax advantages, and can serve as a source of rental income for homeowners who choose to become landlords. Mortgages are available, but home-buyers must satisfy a minimum down payment requirement, and home sales and purchases are subject to lumpy adjustment costs. Lower interest rates, relaxed lending standards, and higher incomes are shown to account for over one-half of the increase in the U.S. house price-rent ratio between 1995 and 2005, and generate the observed pattern of rapidly growing house prices, sluggish rents, increasing homeownership, and rising household indebtedness. The model highlights the importance of accounting for equilibrium interactions between the markets for owned and rented property when analyzing the housing market. These general equilibrium effects can either magnify or reverse the partial equilibrium effects of changes in fundamentals on house prices, rents, and homeownership.

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1 Introduction

The sharp increase and subsequent collapse in U.S. house prices over the past decade has been well documented. Real house prices rose by only 3.7 percent between 1985 and 1995, but increased by 46 percent between 1995 and 2005. In sharp contrast, real rents remained virtually unchanged during the recent increase in house prices, so that in 2006 the house price-rent ratio peaked at approximately forty percent above its level in the year 2000 (Figure 1). The house price-rent ratio is widely used as an indicator of over and undervaluation of the housing market. Yet, despite the widespread use of the price-rent ratio as a key housing market statistic, surprisingly little is known about the theoretical relationship between the price-rent ratio and market fundamentals such as interest rates, income, down payment requirements, and features of the U.S. tax code which favor homeownership over renting and provide sizeable tax subsidies to landlords.

This paper bridges the gap in the existing literature by studying the joint dynamics of endogenously determined house prices and rents in a dynamic equilibrium model of housing tenure choice with fully specified markets for homeownership and rental properties. Our framework is an Aiyagari-Bewley-Huggett style economy with a stochastic life cycle and heterogeneous households who are subject to idiosyncratic earnings shocks. Households derive utility from nondurable consumption and shelter services which are obtained either via renting or through homeownership. Markets are incomplete. Households can partially self-insure earnings risk by accumulating precautionary financial assets: deposits. In addition to deposits, households can hold a non-financial asset: houses. Houses are modeled as durable, indivisible, discrete-sized items which provide housing services, grant access to collateralized borrowing, and can serve as a source of rental income for homeowners who choose to become landlords. The supply of rental housing is thus determined endogenously within the model, as homeowners weigh their utility from shelter space against rental income, taking into account the tax implications of their decisions.\footnote{Using the data from the Property Owners and Managers Survey, Chambers, Garriga, and Schlagenhauf (2008) use micro data evidence to document that a vast majority of U.S. rental property is owned by households, rather than firms. Namely, 86 percent of the U.S. rental property is owned by individual investors (or husband and wife), and fully 94 percent of all rental property is owned by non-institutional investors. The remainder is controlled by real estate corporations, other corporations, non-profit organizations, or church.}

Mortgages are available to finance purchases of housing, but home-buyers must satisfy a minimum down payment requirement. Moreover, home purchases and sales are subject to lumpy transaction costs and the housing stock is subject to depreciation. Households who do not own houses rent housing services in the rental market and do not have access to borrowing or to the preferential tax treatment of owner-occupied housing and rental properties embedded in the U.S. tax code. Both house prices and rents are determined in equilibrium through clearing of housing and rental markets.

The calibrated model is used to study the impact of macroeconomic factors such as incomes, interest rates, and borrowing constraints on the equilibrium price-rent ratio. Our rational expectations model of the housing market demonstrates that the rising incomes, historically low interest rates, and easing of down payment requirements observed in the data can explain about one-half of the increase in U.S. house prices between 1995 and 2005.\footnote{A large body of empirical literature has investigated the relationship between house prices and macroeconomics aggregates. For example, regression analysis by by Englund and Ioannides (1997), Malpezzi (1999), Muellbauer and Murphy (1997), Muellbauer and Murphy (2008), Otrok and Terrones (2008) show that real interest rates, income, income growth, and financial liberalization have a statistically significant effect on real house prices.}
In addition, the model predicts that changes in these factors will have only a small positive effect on equilibrium rents, a result that is consistent with the U.S. data. The price and rent dynamics generated by the model coincide with increases in the homeownership rate and household debt-to-income ratio that are also similar to the actual developments in the U.S. housing market between 1995 and 2005.

The key mechanism in the model generating the run-up in the equilibrium price-rent ratio as macroeconomic conditions change is that the supply and demand of rental property are endogenously determined jointly with the demand for housing. When the mortgage interest rate and required down payment fall, the demand for rental property falls because households switch from renting to owning as homeownership becomes more affordable. At the same time, the supply of rental property increases because investment in rental property becomes more attractive relative to the alternative of holding bank deposits as the interest the dynamics of real house prices.

3 Poterba (1984), Topel and Rosen (1988) and Muellbauer and Murphy (1997) model the relationship between house prices and rents using asset pricing models which predict that the expected return on housing equals (up to a constant) the rate of return on alternative investments. In general, this type of model cannot explain the coexistence of rising house prices and relatively constant or declining rents.

4 The total household debt to disposable income ratio has increased from 80 percent in 1985 to 93 percent in 1995 and to a whopping 141 percent in 2007. At the same time, the U.S. homeownership rate, initially flat at 64 percent between 1983 and 1995, rose to 69 percent by 2005.
rate falls. As a result, the equilibrium rent falls. At the same time, the demand for housing increases because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the supply of housing is fixed, the equilibrium house price rises. An increase in income that is symmetric across all wage groups leads to a roughly proportional increase in house prices and rents, leaving the price-rent ratio unchanged, as it roughly offsets the initial decline in rents while further boosting house prices.

The model provides a number of additional insights about the mechanisms that jointly determine house prices and rents. Both the house price and rent are relatively inelastic with respect to the down payment requirement, so a lessening of credit constraints cannot by itself account for the run-up in the house prices observed in recent years. The key to understanding the small effect of decreases in the required down payment on equilibrium house prices is to realize that changes in equilibrium house prices are primarily driven by shifts in the housing demand by households who find the minimum down payment a binding constraint and, therefore, increase their demand for housing when the lending standards are relaxed. However, relative to the entire market demand for housing, this increase in demand is relatively small, so the resulting house price increase is small. The corresponding increase in household borrowing as credit constraints are relaxed is skewed toward low-income households, as poorer households gain access to mortgage markets and borrow large amounts relative to their labor income to finance their home purchases.

Furthermore, we find that falling interest rates create large increase in house prices, since cheap credit and a low opportunity cost of borrowing boost household willingness and ability to purchase big properties and to finance them using large mortgages. In our economy with a fixed supply of housing, a falling interest rate thus pushes up house prices. As expected, falling interest rates lead to a large increase in household borrowing, since the low interest rate decreases the cost of mortgage financing and, at the same time, lowers the return on household savings. Somewhat surprisingly, a decline in the interest rate reduces the homeownership rate. This happens because as the interest rate falls and equilibrium house prices rise, some low income households are no longer able to afford the minimum down payment on a house.

This paper builds on the growing body of literature which studies housing using quantitative macroeconomics models with heterogenous households. See, for example, Díaz and Luengo-Prado (2008), Chambers, Garriga, and Schlagenhauf (2008), Chambers, Garriga, and Schlagenhauf (2009a), Chambers, Garriga, and Schlagenhauf (2009b), Favilukis, Ludvigson, and Van Nieuwerburgh (2009), Kiyotaki, Michaelides, and Nikolov (2008), Nakajima (2008), Ríos-Rull and Sánchez-Marcos (2008), Ortalo-Magné and Rady (2006), and Iacoviello and Neri (2007). The studies most closely related to ours are Chambers, Garriga and Schlagenhauf (2008, 2009a, 2009b) and Díaz and Luengo-Prado (2008) in terms of the model, and Favilukis, Ludvigson, and Van Nieuwerburgh (2009) and Kiyotaki, Michaelides, and Nikolov (2008) in terms of the theme. Díaz and Luengo-Prado (2008) build a partial equilibrium economy with a number of realistic features such as collateral borrowing, non-convex adjustment costs, taxes, and idiosyncratic earnings risk. However, in their model, housing and rental markets exist only insofar as both house prices and rents follow exogenous processes. Chambers, Garriga and Schlagenhauf (2008, 2009a, 2009b) use the Amer-

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5In the United States, the buy-to-let markets have grown substantially since the mid-1990s (OECD, 2006). The portion of sales attributable to such investors has risen sharply since the late 1990s, reaching around 15 percent of all home purchases in 2004, much higher than the normal 5 percent (Morgan Stanley, 2005).
ican Housing Survey to document that the vast majority of U.S. rental property is owned by households instead of firms, and develop a model where rental property is supplied by households who choose to become landlords as a result of optimal investment strategies. However, the authors allow rents but not house prices to be determined endogenously within their model. This paper adopts the structure of rental markets from Chambers, Garriga, and Schlagenhauf (2009a), but also explicitly models a housing market so that both house prices and rents are determined in an equilibrium. Turning to the dynamics of the price-rent ratio, Kiyotaki, Michaelides, and Nikolov (2008) briefly explore the equilibrium relationship between house prices and rents in a more stylized model where production capital (i.e., factories) can be costlessly transformed into housing structures, and where rent is determined as a factor price of this production capital. The authors, however, focus primarily on the response of welfare to changes in fundamentals. Lastly, Favilukis, Ludvigson, and Van Nieuwerburgh (2009) study the evolution of the price-rent ratio, but their model does not include a rental market. Instead, they impute rent for homeowners using the marginal rate of substitution between consumption and housing. Moreover, the supply of housing in their economy is highly elastic, as the authors abstract from features such as a fixed supply of land or fixed supply of housing.

This paper is organized as follows. In Section 2, we develop a quantitatively rich stochastic life cycle model of the housing market with fully specified household choices with respect to consumption, saving, and homeownership, and provide rationale for our modeling assumptions. Section 3 defines the equilibrium of the economy, while Section 4 describes the model’s calibration and discusses the fit of the benchmark model. In Section 5, we discuss predictions of the benchmark model, and reconcile these with the actual dynamics of house prices and rents in the U.S. data. Section 6 concludes with a discussion of possible extensions and directions for the future research.

2 The Model Economy

The baseline is a small open economy in steady state with inflexible supply of housing and endogenously determined supply of rental properties. The time-invariant house price and rent are determined endogenously within the model through clearing of housing and rental markets.

6 Alternative models that allow for renting typically adopt the representative zero-profit rental firm framework as in Gervais (2002) or Nakajima (2008) in which the supply of rental property is perfectly elastic and, by construction, rents are positively correlated with house prices through a simple arbitrage condition. However, this positive correlation does not always hold in the data. For example, Panel B of Figure 1 shows that there have been protracted periods during which U.S. house prices grew while rents declined. We therefore follow Chambers, Garriga, and Schlagenhauf (2009a) and assume that rental property is supplied by households who choose to become landlords as a result of optimal investment strategies. This approach to modeling the rental market allows the supply of rental property to respond to changes in fundamentals in a non-trivial fashion so that the positive correlation between house prices and rents need not hold. In addition, this framework accounts for the effects of moral hazard in rental markets and the preferential tax treatment of landlords on the supply of rental property.

7 Chambers, Garriga and Schlagenhauf (2008, 2009a, 2009b) have, however, other equilibrium objects, such as interest rates.

8 In a model such as ours with discrete choices, lumpy adjustment costs, and borrowing constraints, the relationship between the MRS, market rent, and the cost of housing is theoretically ambiguous.
2.1 Demography and Endowments

Our framework is an overlapping generations heterogenous-agent economy with incomplete markets and uninsurable idiosyncratic income risk.\(^9\) We follow Heathcote (2005) in modeling the life cycle as a stochastic transition between various labor productivity states that, in a stylized way, also allows households to age. Namely, we use the one-dimensional stochastic state variable, \(w\), to denote the household’s labor endowment. We assume that the process for \(w\) is independently and identically distributed across households, that it takes on values in the \(J\)-dimensional set \(
\{w_1, \ldots, w_J\} = \mathcal{W}\), and that it follows a finite-state Markov chain \(\pi_w(w'|w)\) which is intended to parsimoniously estimate a richer stochastic process. A detailed description of the endowment income process is presented in Section 4.1. In this model, we do not allow for inter-generational transfers of wealth (financial or non-financial) or human capital. Instead, we assume that, upon death, estates are taxed at a 100 percent rate by the government and immediately resold, and young households are born as renters and can accumulate assets only gradually through saving or housing investment.\(^{10}\)

2.2 Preferences

Each household derives utility from consumption of a nondurable good, \(c\), (which is the numeraire) and shelter services, \(s\), provided by residential capital, \(h'\).\(^{11}\) The expected lifetime utility of a household who does not value leisure is

\[
E_t \sum_{t=0}^{\infty} \beta^t \chi(s_t, h_{t+1})u(c_t, s_t), \tag{1}
\]

and \(\beta \in (0, 1)\) is the time-discount factor.

Shelter services may be obtained either via a rental market at a constant price \(\rho\) per unit of housing, or through ownership of housing at a constant price \(q\) per unit of housing.\(^{12}\) A linear technology transforms the housing investment, \(h'\), into housing services, \(s\), so that one unit of housing provides one unit of shelter services. Households cannot rent and be homeowners at the same time, i.e. \(s \leq h'\). Homeowners can, however, become landlords. Namely, as in Chambers, Garriga and Schlagenhauf (2007), homeowners may choose to set \(s < h'\), in which case \((h' - s) =: l\) is leased to renters at rental rate \(\rho\). Being a landlord, however, implies a constant utility loss \(\chi\) caused by the burden of managing and maintaining a rental property. The landlord utility loss is

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\(^{9}\)As discussed in Castaneda, Díaz-Gimenez, and Ríos-Rull (2003), when insurance markets are allowed, the model economy collapses to a representative agent model, as long as the right initial condition holds.

\(^{10}\)This removes the bequest motive from the saving decision. To ensure that such assumption does not lead households to excessively borrow during their lives, we carefully calibrate the model (see Section 4) to ensure that the household borrowing patterns align with the data.

\(^{11}\)We suppress the index of household \(i\) when we describe a typical household. Furthermore, the notation \(x'\) denotes the value of generic variable \(x\) at the end of the period (or equivalently, the instant a new period begins). For example, \(h'\) is the level of housing chosen by an agent after within-period shocks have been realized.

\(^{12}\)The prices \((q, \rho)\) are time-invariant due to the fact that we solve for the steady-state of the economy. For details, see Huggett (1993) or others.
\[ \chi(s, h') = \begin{cases} 
1 - \chi & \text{if } s < h' \\
0 & \text{otherwise.} 
\end{cases} \] 

\(2\)

2.3 Assets and Market Arrangements

There are three types of assets in the economy: residential capital, \(h \geq 0\), deposits, \(d \geq 0\), and collateral debt, \(m \geq 0\), taking on values in sets \(H\), \(D\) and \(M\), respectively. Deposits offer an exogenous return \(r\), while collateral debt (mortgage debt and equity loans) carries an exogenous interest payment \(r^m\). There is no uncertainty about interest rates. Households may alter their individual holdings of the assets \(h, d, m\) to the new levels \(h', d', m'\) at the beginning of period. Homeownership is lumpy in that houses have a minimum size (i.e., \(h_t \geq h\)), and come in discrete sizes (i.e., \(h_t \in \{0, h(1), ..., h(m)\}\)). Agents also make a discrete choice about shelter consumption. Households can rent a small unit of shelter, \(s\), which is smaller than than the minimum house size available for purchase, \(s < h(1)\). To maintain symmetry between shelter sizes available to homeowners and renters, we assume that all other levels of shelter consumption must match a point on the housing grid, so \(s_t \in \{s, h(1), ..., h(m)\}\).

Only households with residential capital (i.e., homeowners) can access to collateralized borrowing. In particular, we assume that, in any given period, a homeowner faces the borrowing constraint

\[ m' \leq (1 - \theta)qh' \]

with a minimum equity requirement, \(\theta > 0\). The equity requirement effectively disposes of free-entry to the housing market, since households interested in buying a house with a market value \(qh'\) must put down at least a fraction \(\theta\) of the value of the house. By the same token, households who wish to sell their house and move to a different size house or become renters must repay all the outstanding debt, since the option of a mortgage default is not available. The accumulated housing equity above the down payment can, however, be used as collateral for home equity loans.\(^{13}\) Moreover, households can access the additional housing equity through costless refinancing. In general, the collateral borrowing is modeled in a spirit of home lines of credit: households with collateral debt are subject to only the per-period interest payments, but do not need to make payments toward the principle.\(^{14}\) There are no other limits to credit availability: regardless of age or income, if a household can pay the down payment, they receive a mortgage.\(^{15}\)

The total housing stock, \(H\), is fully owned by households and its size does not change over time.\(^{16}\) Our set-up with endogenous house prices and inflexible housing supply thus

\(^{13}\)Similarly to Díaz and Luengo-Prado (2008), we abstract from income requirements when purchasing houses. See their paper for further discussion.


\(^{15}\)As discussed in Section 2.1, if the household dies, the government receives the housing asset and resells it right away.

\(^{16}\)Indeed, the available empirical evidence suggests that the housing supply grew in the U.S. metropolitan regions grew only modestly since 1995. Namely, according to the Census data, the median square footage per housing unit increased by 4 percent between 1997 and 2007 in the United States, but most of these increases were observed outside the metropolitan statistical areas. For example, outside MSAs, the median square footage increased by 13 percent between 1997 and 2007. In a sharp contrast, the median square
represents an alternative to a production economy where land – the input factor into the housing production – is in fixed supply.

Buying and selling a house is costly: a fraction of the house value is lost when bought or sold. A household which buys a house pays a transaction cost, $\tau^b$, proportional to the value of the new house (the total buying cost thus equals $\tau^b q h'$). Similarly, a household which sells a house pays a transaction cost, $\tau^s$, proportional to the value of the old house, so selling costs equal $\tau^s q h$. Since there are no realtors in this model, we model the transaction costs as taxes, but interpret them as brokerage fees and other costs related to moving. Importantly, the presence of transactions costs makes housing a relatively illiquid asset, and can generate sizeable inaction regions with regard to the household decision to buy or sell.

Homeowners incur maintenance expenses, which for convenience we take to be immediate. The actual expense depends both upon the value of housing and upon the level of $s$ in relation to $h'$ (e.g., the amount of the property that is rented to other households). Housing which is consumed by the owner depreciates at rate $\delta_o$. We assume that a moral hazard problem exists in the rental market for housing services, namely that housing occupied by a renter depreciates more rapidly than owner occupied housing. This problem arises because renters decide how intensely to utilize a house but may not actually pay the resulting cost, which creates an incentive to overutilize the property. The depreciation rate for rented property is $\delta_r$, and $\delta_r > \delta_o$. Thus, current total maintenance costs facing an agent who has just chosen housing equal to $h'$ are given by

$$M(h', s) = I_{h' \neq 0}[\delta_0 s + I_{h' > s}\delta_r(h' - s)],$$

with the binary indicator $I_{h' \neq 0}$ denoting that a household is a homeowner, and $I_{h' > s}$ indicating that a household is also a landlord.

### 2.4 The Government

We follow Díaz and Luengo-Prado (2008) in modeling a tax system with a preferential tax treatment of owner-occupied housing that mimics the U.S. system in a stylized way. Namely, in addition to the taxation of household labor and asset income, the government imposes a proportional property tax on housing which is fully deductible from income taxes, and allows deductions for interest payments on collateral debt (mortgages and home equity). As in the U.S. tax code, the imputed rental value of owner-occupied housing is excluded from taxable income. We expand on the tax treatment of rental property in existing models of the housing market by allowing landlords to deduct depreciation of the rental property from their taxable income. For simplicity, we assume proportional income taxation at the rate $\tau^y$. We do not require a balanced budget every period.

The total taxable income is thus defined as

$$\tilde{y} = w + rd + I_{h' \neq 0} \left[ -\tau^m r^m m - \tau^b q h' \right] + I_{h' > s} [\rho (h' - s) - \tau^{LL} q (h' - s) - \delta_r q (h' - s)],$$

where $w + rd$ represents household labor income plus earned interest. The first term in

footage per housing unit in MSA cities decreased at -0.2 percent between 1997 and 2007, while in MSA suburbs the square footage per house grew by 1.5 percent over the period. Moreover, the increases in the aggregate housing supply coincided with population growth which increased the U.S. population increased by 12.5 percent between 1997 and 2007 (4.7 percent between 2000 and 2005).
brackets represents the tax deduction received by homeowners, where $\tau^m \tau^m m$ is the mortgage interest deduction, and $\tau^h q h'$ is the fully deductible property tax payment made by the household. The next term in brackets represents the taxable rental income of landlords, which equals total rents received, $\rho (h' - s)$, minus the tax deductions available to landlords. The term $\tau^{LL} q (h' - s)$ represents the tax deduction for depreciation of the rental property, where $\tau^{LL}$ represents the fraction of the total value of the rental property that is tax deductible in each year. The final term that determines taxable rental income, $\delta_r q (h' - s)$, represents tax deductible maintenance expenses. If the tax deductions for the rental property exceed rental income, so $\rho (h' - s) < \tau^{LL} q (h' - s) + \delta_r q (h' - s)$, then rental losses will reduce the households’ tax liability by offsetting income from wages and interest, $w + rd$.

At this point it is useful to discuss the current U.S. tax treatment of landlords and explain how the key features of the tax code are incorporated into our model. Landlords must pay income taxes on rental income. However, landlords are permitted to deduct many different expenses associated with operating a rental property from their gross rental income when determining the amount of rental income that is subject to income taxes. Among the major tax deductible rental expenditures incorporated into our model are mortgage interest payments, property taxes paid on the rental property, depreciation of the rental structure, and maintenance expenditures. The amount of the depreciation deduction is specified in the U.S. tax code, and we discuss the exact depreciation rate used in our model in Section 4. In addition, landlords who meet a minimum standard of involvement with their rental property may use rental losses to offset income earned from sources other than real estate.

2.5 Equilibrium

Each period the economy-wide state is a measure of households, $\lambda$, defined over $\mathcal{B}$, an appropriate family of subsets of $\{D \times M \times H \times W\}$. As far as each individual household is concerned, the state variables are the realization of the household-specific shock, $w$, the current asset position, $(d, m, h)$, and the aggregate state, $\lambda$. Let $x = (w, d, m, h)$. In a steady state, the measure of households, $\lambda$, remains time-invariant, implying that household’s state variable is simply the vector $x$.

2.5.1 Timing of events

A household starts any given period $t$ with a stock of residential capital, $h \geq 0$, deposits, $d \geq 0$, and collateral debt (mortgage debt and equity loans), $m \geq 0$. Households observe the idiosyncratic earnings shocks, $w$, and – given the current prices $(q, \rho)$ – choose new levels of nondurable consumption, $c$, shelter, $s$, as well as their new asset position $(h', d', m')$. Namely, homeowners ($h > 0$) choose whether to adjust the size of their house (so that $h' \neq h$), and whether or not to become a landlord ($h' > s$). Households currently renting ($h = 0$) choose

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17 Other expenses that are tax deductible but not incorporated in our model are expenses related to advertising, travel to the rental property, commissions, insurance, legal and professional fees, management fees, supplies, and utilities. See IRS publication 527 for details on the tax treatment of residential rental property.

18 A maximum of $25,000 in rental property losses can be used to offset income from other sources, and this deduction is phased out between $100,000 and $150,000 of income. In our stylized model we abstract away from the $25,000 limit and we do not incorporate the phasing out of this deduction for high income households into our model of the tax system.
whether to continue to rent \((h' = 0)\), or enter the housing market \((h' > 0)\). If a household enters the housing market, they can become a landlord. Households receive interest on deposits, \(r\), and pay interest on collateral debt, \(r^m\). There is no uncertainty about interest rates. Landlords receive rent payments from their tenants, \(\rho(h' - s)\). Households pay taxes and homeowners cover maintenance cost, \(qM(h', s)\). Households which are buying or selling a house \((h' \neq h)\) incur transaction cost \(\tau^b qh'\) and \(\tau^s qh\), respectively. In particular, homeowners who increase or decrease the size of their homes pay both the buying and selling fees. Renters who newly become homeowners incur buying fees only. Similarly, former homeowners who sell their property and become homeowners incur selling fees only.

### 2.5.2 The Dynamic Programming Problem

Each period, a household whose state is \(x = (w, d, m, h)\) solves the dynamic program:

\[
v(w, d, m, h) = \max_{c, s, h', d', m'} \chi(s, h)u(c, s) + \beta \sum_{w' \in W} \pi(w'|w)v(w', d', m', h')
\]

subject to the constraints

\[
c + \rho (s - h') + d' - m' + q(h' - h) + I^s \tau^s qh + I^b = \tau^b qh'
\]

\[
\leq w + (1 + r)d - (1 + r^m) m - \tau^b \tilde{y} - \tau^b qh' - qM(h', s)
\]

\[
m' \leq (1 - \theta) qh'
\]

\[
m' \geq 0
\]

\[
d' \geq 0
\]

\[
h' \geq s
\]

by choosing consumption, \(c\), and shelter, \(s\), as well as current levels of housing investment, \(h'\), deposits, \(d'\), and collateral debt, \(m'\). \(\rho (s - h')\) represents either a rental payment by renters (i.e., households with \(h' = 0\)), or the rental income received by landlords (i.e., households with \(h' > s\)). \(q(h' - h)\) captures the cost of new housing investment over its current value. \(\tau^s qh\) represents the transaction fees incurred when a property is sold (i.e., \(I^s = 1\) if \(h_t \neq h_{t-1} > 0\); zero otherwise), while \(\tau^b qh'\) captures the fees incurred when a new property is purchased (i.e., \(I^b = 1\) if \(0 < h_t \neq h_{t-1} < 0\); zero otherwise). \(w\) represents the household income and follows a process \(\pi_w(w|w_{t-1})\) described in Section 2.1. \(rd\) and \(r^m m\) capture the interest income on deposits and the mortgage payment, respectively. \(\tau^b \tilde{y}\) is the total income tax paid of the taxable income \(\tilde{y}\) in Equation 5. \(\tau^b qh'\) describes the property tax paid by homeowners. Finally, \(qM(h', s)\) represents the maintenance expenses for homeowners in Equation 4.

### 3 Definition of a Stationary Equilibrium

A steady state equilibrium for the baseline economy is a household value function, \(v(x)\), a household policy \(\{c(x), s(x), d'(x), m'(x), h'(x)\}\), a probability measure of agents over the individual states, \(\lambda\), and price vector \((q, \rho)\) satisfying:

1. \(c(x), s(x), d'(x), m'(x), \text{and } h'(x)\) are optimal decision rules to the households’ decision
problem, given prices \( q \) and \( \rho \)

2. Markets clear:

   (a) Housing market clearing: \( \int h'(x)d\lambda = H \), where \( H \) is fixed

   (b) Rental market clearing: \( \int (h'(x) - s(x))d\lambda = 0 \),

   where integrals are defined over the state space \( \{ D \times M \times H \times W \} \).

3. \( \lambda \) is a stationary probability measure.

4 Calibration

The method of simulated moments is used to calibrate the model based on cross-sectional patterns of income, wealth, homeownership, and landlord characteristics. Table 1 summarizes parameters which were drawn from other studies or were calculated directly from the data. Table 2 contains four estimated parameters based on the moments described in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation ( \rho_w )</td>
<td>0.90</td>
</tr>
<tr>
<td>Standard Deviation ( \sigma_w )</td>
<td>0.20</td>
</tr>
<tr>
<td>Risk Aversion ( \sigma )</td>
<td>2.00</td>
</tr>
<tr>
<td>Down Payment Requirement ( \theta )</td>
<td>0.20</td>
</tr>
<tr>
<td>Selling Cost ( \tau^s )</td>
<td>0.07</td>
</tr>
<tr>
<td>Buying Cost ( \tau^b )</td>
<td>0.025</td>
</tr>
<tr>
<td>Risk-free Interest Rate ( r )</td>
<td>0.04</td>
</tr>
<tr>
<td>Spread ( \kappa )</td>
<td>0.015</td>
</tr>
<tr>
<td>Depreciation Rate for Homeowner-Occupiers ( \delta_0 )</td>
<td>0.025</td>
</tr>
<tr>
<td>Property Tax Rate ( \tau^h )</td>
<td>0.01</td>
</tr>
<tr>
<td>Mortgage Deductibility Rate ( \tau^m )</td>
<td>1.00</td>
</tr>
<tr>
<td>Deductibility Rate for Depreciation of Rental Property ( \tau^{LL} )</td>
<td>0.023</td>
</tr>
<tr>
<td>Income Tax ( \tau^y )</td>
<td>0.20</td>
</tr>
</tbody>
</table>

4.1 The Endowment Process

A time period in the model is one year. As discussed previously, we consider a version of the stochastic-aging economy that is designed to capture the idea that liquidity constraints may be most important for younger individuals who are at the bottom of an upward-sloping lifetime earnings profile. We follow Heathcote (2005) and allow households to transit from state \( w \) via two mechanisms: (i) aging and (ii) productivity shocks, where the events of aging
Table 2: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor $\beta$</td>
<td>0.959</td>
</tr>
<tr>
<td>Consumption Share $\alpha$</td>
<td>0.720</td>
</tr>
<tr>
<td>Depreciation of Rental Property $\delta_r$</td>
<td>0.037</td>
</tr>
<tr>
<td>Landlord Utility Loss $\chi$</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Table 3: Calibration Targets

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home-ownership rate</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>Landlord rate</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Imputed rent-to-wage ratio</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Fraction of homeowners with collateral debt</td>
<td>0.65</td>
<td>0.64</td>
</tr>
</tbody>
</table>

and receiving productivity shocks are assumed to be mutually exclusive. The probability of transiting from a state $w_j$ via aging is equal to $\phi_j = 1/(p_j L)$, where $p_j$ is the fraction of population with productivity $w_j$ in the ergodic distribution over the support $W$, and $L$ is a constant equal to the expected lifetime. Similarly, the conditional probability of transiting from a working-age state $w_j$ to a working-age state $w_i$ due to a productivity shock is defined as $P(w_i|w_j)$. The overall probability of moving from state $j$ to state $i$, denoted by $\pi_{ji}$, is therefore equal to the probability of transition from $j$ to $i$ via aging, plus the probability of transition from $j$ to $i$ via a productivity shock, conditional on not aging, so that

$$\Pi = \begin{bmatrix} 0 & \phi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{J-1} \\ \phi_J & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} (1 - \phi_1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (1 - \phi_{J-1}) & 0 \\ 0 & 0 & 0 & (1 - \phi_J) \end{bmatrix} P. \tag{11}$$

The fractions $p_j$ are the solutions to the system of equations $p = p\Pi$.

To calibrate the stochastic aging economy, we assume that households live, on average, 50 periods (e.g., $L = 50$). In terms of the process for household productivity, many papers in the quantitative macroeconomics literature adopt simple AR(1) specification to capture the earnings dynamics for working-age households that is characterized by the serial correlation coefficient, $\rho_w$, and the standard deviation of the innovation term, $\sigma_w$.$^{19}$ Using the data from the Panel Study of Income Dynamics (PSID), work by Card (1991), Hubbard, Skinner, and Zeldes (1995) and Heathcote, Storesletten and Violante (2003) indicates a $\rho_w$ in the range 0.88 to 0.96, and a $\sigma_w$ in the range 0.12 to 0.25. For the purposes of this paper, we set $\rho_w$ and $\sigma_w$ to 0.90 and 0.20, respectively, and follow Tauchen (1986) to approximate an otherwise continuous process with a discrete number (7) states.

$^{19}$ Heathcote (2005) discusses alternatives to the AR(1) specification in a technical appendix which is available on the Review of Economic Studies web site.
4.2 Preferences

We assume that preferences over the consumption of goods and housing services can be represented by the following utility function,

\[ u(c, s) = \left( \frac{c^{\alpha} s^{1-\alpha}}{1 - \sigma} \right)^{1-\sigma}. \]  

(12)

To characterize household preferences, we must choose values for four parameters. The risk aversion parameter, \( \sigma \), is set to 2. The discount factor (\( \beta \)), Cobb-Douglas share parameter (\( \alpha \)), and landlord utility loss parameter (\( \chi \)) are calibrated. The share parameter \( \alpha \) affects the allocation of income between the two expenditure components. Using the data from 1980, 1990, and 2000 Decennial Census of Housing, Davis and Ortalo-Magné (2008) estimate the share of expenditures on housing services by renters to be roughly 0.25, and find that the share has been constant across time and MSA regions. We thus calibrate \( \alpha \) to match this share. Moreover, the discount factor \( \beta \) is calibrated to match the fraction of owner-occupiers with collateral debt. According to data from the 1994-1998 American Housing Survey (ASH), approximately 65 percent of homeowners report collateral debt balances. The parameter \( \chi \) that characterizes the utility loss for landlords in equation (2) is set to match the average fraction of homeowners (0.66) in the United States between 1995 and 2005.

4.3 Market Arrangements

In the benchmark model, a minimum down payment of 20 percent is required to purchase a home.\(^{21}\) With regard to the transaction costs, Gruber and Martin (2003), using the data from the Consumption Expenditure Survey (CE), document that selling cost for housing can be up to 7 percent, while buying costs are around 2.5 percent. We use the authors’ estimates and set \( \tau^s = 0.025 \) and \( \tau^b = 0.07 \).

To calibrate the interest rates on deposits and collateral debt, we follow Díaz and Luengo-Prado (2008) and assume that the collateral debt is associated with an interest rate \( r^m = r + \kappa \), where \( \kappa > 0 \) represents the spread between the two rates. Based on data from the Federal Reserve Statistical Release, the average spread between the nominal interest rate on a 30-year fixed-rate conventional home mortgage and the interest rate on nominal 30-year constant maturity Treasury (or T-bond) between 1977 and 2008 is 1.5 percent, so that \( \kappa \) is set to 0.015.\(^{22}\) For consistency, we use the interest rate on the same 30-year constant maturity T-bonds to represent the interest rate on deposits, \( r \). The average rate for the period between 1977 and 2008 fluctuated between -2.23 and 8.04, with an average for the period of 3.76.\(^{23}\) We thus set the real interest rate to 4 percent so that \( r = 0.04 \).

\(^{20}\)The discount pattern \( \beta \) governs household borrowing behavior in our model. Since deceased agents in our model are replaced by newborn descendants who do not, however, inherit the asset positions of the dead, we calibrate \( \beta \) to ensure that households do not borrow excessively and to generate a realistic borrowing behavior of households in our model economy.

\(^{21}\)Using the American Housing Survey 1993, Chambers, Garriga and Schlagenhauf document that the average down payment is approximately 20 percent.

\(^{22}\)The spread has fluctuated between 0.73 and 3.32 percent between years 1977 and 2008. The average spread for the period is 1.59 percentage points while the median spread is 1.5. For the data used to construct the spread, see Federal Reserve Statistical Release, H15, Selected Interest Rates.

\(^{23}\)The median interest rate for the period is 3.83.
To parametrize the maintenance cost function \( M(h', s) \) in equation (4), we follow Harding, Rosenthal, and Sirmans (2007) who estimate the depreciation rate for housing units used as shelter between 2.5 and 3 percent. We thus set \( \delta_0 = 0.025 \) and estimate the depreciation rate of rental property, \( \delta_r \), so that the model delivers a landlord rate and homeownership rate comparable to that in the U.S. economy. Chambers, Garriga and Schlagenhauf (2008) use the American Housing Survey data to compute the fraction of homeowners who claim to receive rental income. The authors find that approximately 10 percent of the sampled homeowners receive rental income. We use the authors’ estimate of the “landlord rate” to help identify \( \delta_r \).

### 4.4 Taxes

Using data from the 2007 American Community Survey, Díaz and Luengo-Prado (2009) compute the median property tax rate for the median house value and report a housing property tax rate of 0.95 percent. Moreover, the authors, using information from TAXSIM, the deduction percentage for interest payments of 0.9. We thus set \( \tau^h = 0.01 \), and allow mortgages to be fully deductible so that \( \tau^m = 1 \). The U.S. tax code assumes that a rental structure depreciates over a 27.5 year horizon, which implies an annual depreciation rate of 3.63 percent. However, only structures are depreciable for tax purposes, and the value of a house in our model includes both the value of the structure and the land that the house is situated on. Davis and Heathcote (2007) find that on average, land accounts for 36 percent of the value of a house in the U.S. between 1975 and 2006. Based on their findings, we set the depreciation rate of rental property for tax purposes to \( \tau^{LL} = (1 - .36) \times .0363 = .023 \). Lastly, we follow Díaz and Luengo-Prado (2008) and Prescott (2004) and set the income tax rate, \( \tau^y \), to 0.20.

### 4.5 Calibration Results

#### 4.5.1 Moment Conditions

As discussed previously, our calibration is designed to match the U.S. homeownership rate (0.66), the fraction of households who receive income from rental property (0.10), the fraction of homeowners with collateral debt (0.65), and the ratio of housing services expenditures to wages (0.25). Targeting the homeownership and landlord moments implies that we are also implicitly targeting the fraction of households who are renters (0.34) and owner-occupiers (0.56) because the landlord, renter, and owner-occupier categories are mutually exclusive and collectively exhaustive. As can be seen in Table 3, we match these moments well.

Table 4 reports several other important statistics generated by the model and compares these with the estimates that are either drawn from other studies or the official AHS tables, or are computed from the 2007 Survey of Consumer Finances. Appendix A describes how we compute the SCF statistics in the data. As can be seen in the table, the average net worth-to-income ratio for homeowners, where net worth is defined as the sum of deposits and housing wealth net of collateral debt, generated by the model is 2.9, which is close to the 2007 SCF estimate of 3.2. The house value-to-income ratio for homeowners of 3.64 lies between the comparable estimates in the AHS and SCF: 3.1 and 4.0, respectively. The loan-to-value ratio for homeowners of 1.19 aligns nicely with the 2007 SCF estimate of 1.16. At the same time, the loan-to-value ratio for homeowners of 0.31 matches closely the 2007
Table 4: Other Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net worth to total income ratio for homeowners</td>
<td>2.94</td>
<td>3.17</td>
<td>SCF 2007</td>
</tr>
<tr>
<td>Housing value to total income ratio for homeowners</td>
<td>3.64</td>
<td>4.02 / 3.1</td>
<td>SCF 2007 / AHS 2005</td>
</tr>
<tr>
<td>Loan to total income ratio for homeowners</td>
<td>1.19</td>
<td>1.16</td>
<td>SCF 2007</td>
</tr>
<tr>
<td>Loan to value ratio for homeowners</td>
<td>0.31</td>
<td>0.28 / 0.55</td>
<td>SCF 2007 / AHS 2005</td>
</tr>
<tr>
<td>Rental income receipts to income ratio for landlords</td>
<td>0.28</td>
<td>0.31</td>
<td>AHS 2005</td>
</tr>
<tr>
<td>House price-rent ratio</td>
<td>11.4</td>
<td>8 - 15.5</td>
<td>Various studies</td>
</tr>
</tbody>
</table>

Table 5: Distribution of Households Across House Sizes

<table>
<thead>
<tr>
<th>Housing Owned (h)</th>
<th>Room (s)</th>
<th>Small shelter-size (s)</th>
<th>Medium shelter-size (s)</th>
<th>Large shelter-size (s)</th>
<th>% HHs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renter (h = 0)</td>
<td>67.90</td>
<td>32.10</td>
<td>0.00</td>
<td>0.00</td>
<td>33.75</td>
</tr>
<tr>
<td>Small-size property</td>
<td>0.63</td>
<td>99.37</td>
<td>0.00</td>
<td>0.00</td>
<td>13.94</td>
</tr>
<tr>
<td>Medium-size property</td>
<td>1.57</td>
<td>6.52</td>
<td>91.90</td>
<td>0.00</td>
<td>46.74</td>
</tr>
<tr>
<td>Large-size property</td>
<td>0.00</td>
<td>0.58</td>
<td>99.25</td>
<td>0.14</td>
<td>5.56</td>
</tr>
<tr>
<td>% HHs</td>
<td>23.74</td>
<td>27.77</td>
<td>48.48</td>
<td>0.01</td>
<td>100.00</td>
</tr>
</tbody>
</table>

SCF estimate of 0.28, but both the model and the 2007 SCF estimate understate the 2005 AHS statistics of 0.55. The model also predicts a ratio of rental income to total income for landlords at 0.28, which is close to the ratio of 0.31 estimated in the 2005 AHS. Finally, the model generates a house price-rent ratio of roughly 11.6. The U.S. Department of Housing and Urban Development and the U.S. Census Bureau report a price-rent ratio of 10 in the 2001 Residential Finance Survey (chapter 4, Table 4-2). Garner and Verbrugge (2009), using Consumer Expenditure Survey (CES) data drawn from five cities over the years 1982 – 2002, report that the house price to rent ratio ranges from 8 to 15.5 with a mean of approximately 12. The house price rent ratio of 11.6 generated by the model therefore falls well within the range of recent estimates based on U.S. data. Overall, the ability of the model to fit a number of key moments that were not targeted during the calibration is encouraging.

4.5.2 Cross-sectional Implications of the Model

There are twelve discrete shelter sizes in our model economy: eleven self-standing discrete-size housing structures that can be purchased in the housing market, and a very small living space that can be rented out but is not available for sale. Discreteness in housing captures the idea that housing units typically come in discrete sizes, such as one bedroom, two bedroom, or four bedroom. At the same time, the smallest-size shelter unit, which we call a “room,” captures the idea that agents can also rent a very small living space that is not, however, available for sale so that, for example, a person can share a room with a roommate or can rent

24 The cities included in this analysis are Chicago, Houston, Los Angeles, New York, and Philadelphia.
Table 6: Distribution of Landlords by Labor Income

<table>
<thead>
<tr>
<th>Income group</th>
<th>% Landlords</th>
<th>% Total Rental Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>3.32</td>
<td>1.7</td>
</tr>
<tr>
<td>Group 2</td>
<td>15.02</td>
<td>10.2</td>
</tr>
<tr>
<td>Group 3</td>
<td>33.85</td>
<td>20.7</td>
</tr>
<tr>
<td>Group 4</td>
<td>15.44</td>
<td>20.8</td>
</tr>
<tr>
<td>Group 5</td>
<td>14.47</td>
<td>20.8</td>
</tr>
<tr>
<td>Group 6</td>
<td>12.32</td>
<td>17.7</td>
</tr>
<tr>
<td>Group 7</td>
<td>5.58</td>
<td>7.8</td>
</tr>
</tbody>
</table>

a room while sharing the kitchen. For clarity of exposition, we divide the properties owned by households into three groups called small, medium, and large size properties. The small properties represent starter homes, while medium sized properties are owned by agents who represent the average households in terms of wealth and income. Finally, large properties are in general used for investment, as these often serve as rental units.

Table 5 shows the relationship between units of housing owned and units of shelter consumed. As can be seen in the table, 68 percent of renters live in a room, while the remaining 32 percent of renters inhabit the small size house. The renters are typically hand-to-mouth agents who are at the bottom of the wealth distribution and have savings that are below the minimum down payment requirement for the smallest house.

The renters lease housing services from homeowners who choose to become landlords by consuming less shelter than they currently own. The landlords are typically highly leveraged and often low earnings households who partially lease out their homes to boost their income level. Table 5 shows that 8.1 percent of the owners of medium sized properties are landlords, and supply 39 percent of the total amount of shelter that is rented. Virtually all owners of large properties are landlords (99.9 percent). Although these households comprise only 5.6 percent of the population, they supply 61 percent of the shelter services that are obtained through the rental market. Table 6 shows that low and middle income agents account for a large fraction of the landlords in the model economy. This prediction is consistent with the findings of Chambers, Garriga and Schlagenhauf (2007) who, using the 1996 Property Owners and Managers Survey, find that 25 percent of households receiving rental income are low-income households with annual earnings below $30,000, compared to 30 percent of high-income households with annual earnings over $100,000 (see their Table 4).

Owner-occupiers consume all of the housing services provided by their property. The vast majority of owner-occupiers are divided between the small and medium house sizes and represent the average household in terms of earnings and financial wealth. The remaining owner occupiers live in large properties, represent only 0.1 percent of the population, and are very rich people with medium to high wages.

In general, homeownership is preferred to renting. Households who can afford a down payment on a house typically enter the housing market and become homeowners. Interestingly, the option to become a landlord plays an important role in our model economy, as rental income helps low and medium income households who are typically highly leveraged to keep up with homeownership expenses and payments. For example, the average owner-occupier of a medium-size house has a large amount of financial wealth and receives
a wage endowment that is roughly 30 percent higher than the economy’s average, while an average landlord who owns the same size house earns a wage that is 8 percent lower than average, and is in debt. The option to become a landlord is, however, also popular among rich homeowners who purchases sizeable properties as an investment.

5 What Explains the Changes in the Price-Rent Ratio?

The estimated model is employed to analyze the observed changes in the house prices, rents, and the price-to-rent ratio since mid-1990s. We first study the model’s predictions about the responsiveness of house prices and rents, and the price-rent ratio, to changes in interest rates, borrowing constraints, and household incomes. Then we consider the combined effects of these macroeconomic factors on the housing market equilibrium. As a cross-check, we also study the model’s implications for the homeownership rate, loan-to-income, and loan-to-value ratios.

5.1 Relaxation of Down Payment Requirements

Since the early 1990s, a number of developments have occurred with respect to the financing of housing investment. Financial innovations such as interest-only loans and combo mortgages provided households with greater choices in mortgage debt financing and significantly reduced down payment requirements. Moreover, policies enacted by the Clinton and Bush Administrations targeted lowering of the down payment requirement to increase households’ access to mortgage financing and to generate additional first time home buyers.\textsuperscript{25} As a result, the average down payment declined from about 20 percent in the mid-1990s to 15 percent in the 2000s.\textsuperscript{26}

Figure 2 illustrates the impact of variation in the minimum down payment requirement, $\theta$, on equilibrium housing market outcomes. As the down payment requirement falls from 40 percent to 15 percent, both the equilibrium house price and rent increase by roughly 8 percent, so the price-rent ratio remains virtually unchanged. A reduction in the average down payment requirement in line with the recent U.S. experience from 0.20 to 0.15, leads to a 5.8 percent increase in the house price and a 5.3 percent increase in rent. Since both the house price and rent are relatively inelastic with respect to the down payment requirement, a lessening of credit constraints cannot by itself explain the run-up in house prices observed

\textsuperscript{25}The Clinton Administration enacted policies through the Federal Home Administration (FHA) to lower the downpayment requirements with mortgage insured loans, while the Bush Administration developed the Zero-Downpayment Initiative for FHA to generate additional first-time buyers.

\textsuperscript{26}Chambers, Garriga and Schlagenhauf (2008), using the data from the American Housing Survey (AHS), document that between 1995 and 2003 the average downpayment for FHA loans declined from 21.6 percent in 1995 to 13.8 percent in 1999 before rising again to 16.3 percent in 2003. At the same time, the average downpayment on a non-FHA loan has decreased from 29.8 percent in 1995 to 24.1 percent by 2003. Chomsiengphet and Pennington-Cross (2006) document similar trends in the subprime lending markets. In addition, the fraction of households with a loan to value ratio greater than 90 percent rose from 10 percent in 1990 to 25 percent by 1995 before retracting slightly to 18 percent in 2005, according to the Federal Finance Board. More generally, the down payment requirements were significantly relaxed during the periods 1995-1998 and 2001-2004, although the financial markets tightened slightly temporarily in the wake of the 1998 Asian crisis.
in recent years.\textsuperscript{27}

That said, lower down payment requirements lead to large increases in the homeownership rate as poorer households gain access to mortgage markets and borrow larger amounts to finance home purchases. When $\theta$ falls from 0.40 to 0.15, the homeownership rate increases from 66 percent to 80 percent. At the same time, the loan-to-wage ratio jumps up from 0.7 to 1.4, while the fraction of homeowners in debt rises from 53 percent to 64 percent. The increase in household borrowing is skewed toward low-earnings households, as relatively more low-wage households enter the housing market.

The key to understanding the small effect of decreases in the required down payment on house prices is to realize that the housing market responses are primarily driven by households who find the minimum down payment to be a binding constraint. Decreasing the down payment requirement thus primarily affects low-income, low-savings households who wish to become homeowners but are unable to because of the high equity requirement. As a result, when the down payment requirement falls from 20 percent to 15 percent, the proportion of renters decreases from 34 to 20 percent as previously credit constrained households switch from renting to owning a small-sized house (Table 7).

\textsuperscript{27}The U.S. real house prices rose by 11 percent while the real rents grew by 3 percent between 1995 and 2000. During this period, the real deposit rate on 10 year constant maturity T-bonds oscillated in a relatively narrow range between 3 and 4 percent.
Table 7: The Distribution of Owned Housing Under Different Downpayment Requirements

<table>
<thead>
<tr>
<th>House Size</th>
<th>20% Downpayment</th>
<th>15% Downpayment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Households</td>
<td>% Housing Stock</td>
</tr>
<tr>
<td>Renter</td>
<td>33.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Small-size property</td>
<td>13.9</td>
<td>11.4</td>
</tr>
<tr>
<td>Medium-size property</td>
<td>46.7</td>
<td>63.7</td>
</tr>
<tr>
<td>Large-size property</td>
<td>5.5</td>
<td>24.9</td>
</tr>
</tbody>
</table>

Table 8: The Partial and Equilibrium Effects of a Reduction in the Equity Requirement to 15%

<table>
<thead>
<tr>
<th></th>
<th>Baseline (1)</th>
<th>15% Equity Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Prices</td>
<td>Equilibrium Prices (3)</td>
</tr>
<tr>
<td>House Price</td>
<td>2.55</td>
<td>2.55</td>
</tr>
<tr>
<td>Rent</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Share of Homeowners</td>
<td>0.66</td>
<td>0.81</td>
</tr>
<tr>
<td>Share of Renters</td>
<td>0.34</td>
<td>0.19</td>
</tr>
<tr>
<td>Share of Landlords</td>
<td>0.10</td>
<td>0.11</td>
</tr>
<tr>
<td>Share of Owner-Occupiers</td>
<td>0.56</td>
<td>0.70</td>
</tr>
<tr>
<td>Share of Homeowners in Debt</td>
<td>0.64</td>
<td>0.69</td>
</tr>
</tbody>
</table>
Table 8 provides further details on how changes in the down payment requirements affect the housing market. Column (2) shows the impact of a decline in down payment requirement from 20 to 15 percent under the restriction that house prices and rents are not allowed to change (i.e., both house prices and rents are held fixed at their equilibrium values from the baseline version of the model). Column (3) reports the impact of a decrease on the down payment requirement when both house prices and rents are allowed to adjust to clear the housing and rental markets at the lower down payment requirement. As can be seen in the table, since house prices changed relatively little, the share of landlords among homeowners remains broadly unchanged (Columns (2) and (3)) relative to the baseline case. In addition, while a number of households switch from renting to homeownership, only a small increase in the house price is needed to achieve the redistribution of properties to accommodate the entrants to the housing market. For example, while the percentage of households who own small-size houses jumps up because of the influx of new homeowners into the housing market as the down payment requirement falls from 4 to 2 percent, the fraction of households owning the medium size-property declines (Table 7). In addition, fewer households purchase the large houses which are exclusively occupied by landlords.

Our results are consistent with several recent studies which document the positive correlation between the size of the down payment requirement and homeownership (e.g., Chambers, Garriga, and Schlagenhau (2008), Diaz and Luengo-Prado (2008), and Ortalo-Magné and Rady (2006)). These studies suggest that, while the financial sector innovations have minimal impact for existing homeowners, lower down payment requirements do affect households who are excluded from the housing market due to a high down payment constraint. The authors suggest that when down payment requirements are relaxed, the initially excluded households enter the housing market and the homeownership rate rises. This mechanism is supported by the empirical findings in Ortalo-Magné and Rady (1999) who document that decreases in the down payment requirements in England and Wales after the financial liberalization of the early 1980s were associated with unprecedented increases in the homeownership for young households. Using regression analysis, Muellbauer and Murphy (1990), Muellbauer and Murphy (1997) show that while the current income and short-term demographic changes were the most important factors behind the U.K. house price boom during the 1980s, but that the increase in homeownership was mostly due to the credit market liberalization and the extrapolative price expectations.

In summary, the model clearly indicates that in the absence of changes in other factors, a relaxation of borrowing constraints cannot by itself account for the magnitude of the recent increase in the price-rent ratio. With this result in mind, the next sections of the paper examine the impact of changes in the interest rate and income on the equilibrium price-rent ratio.

5.2 Changes in the Interest Rate

Figure 3 shows the evolution of the real contract and effective mortgage rates on conventional single-family mortgages in the United States between 1985 and 2005.\textsuperscript{28} As can be seen in the figure, the real mortgage rate for residential property oscillated around the 5 percent mark between 1990 and 1997, but started to fall following the 1998 Asian Financial Crisis.

\textsuperscript{28}The effective rate represents the sum of the contract rate and the discounted initial fees and charges. The estimates provided by the Federal Housing Financing Board.
before 2.5 percent in 2005.29

Figure 4 captures the impact of changes in the real risk-free rate, \( r \), on the steady state equilibrium of the housing market. Changes in the risk-free rate interest rate in our model directly translate into changes in the mortgage interest rate because the mortgage interest rate is determined by a constant markup, \( \kappa \), over the risk-free rate. Therefore, changes in the risk-free interest rate affect both the cost of borrowing and the rate of return on saving. As can be seen in the figure, when the real interest rate falls from 6 percent to 1 percent, the equilibrium house price increases by 32 percent, the equilibrium rent decreases by 15 percent, and the price-rent ratio increases by 54 percent from 9.9 to 15.2. When the interest rate declines from 4 percent to 2 percent – a decrease broadly consistent with the actual decline between 1995 and 2005 – the house price level rises by 17 percent, the rent falls by 1.8 percent, and the price-rent ratio rises by 19 percent from 11.4 to 13.6.

As expected, a lower interest rate leads to a large increase in household borrowing, since the low interest rate decreases the cost of mortgage financing and, at the same time, lowers the rate of return on household savings. The average loan-to-wage ratio increases from 0.7 to 3.3 when the interest rate permanently declines from 6 percent to 1 percent, while the fraction of homeowners with mortgage debt rises from 41 percent to 85 percent. For a decline in the interest rate from 4 percent to 2 percent, the loan-to-wage ratio roughly doubles, rising from 1.4 to 2.7.

Turning to the rental market, lower interest rates increase the supply of rental property

29 The mortgage spread, defined as the difference between the real mortgage rate on a 30-year conventional fixed-rate mortgage and the interest rate on a 30-year constant maturity Treasury, fluctuated in a relatively narrow range between 1 and 2 percent since 1995, although the mark-up fell temporarily below one percent between 1991 and 1993.
because, holding the rent fixed, a lower interest rate increases the rate of return to investing in rental property for landlords with mortgages. In addition, investing in rental properties also becomes more attractive relative to the alternative of holding bank deposits. The increase in rental supply decreases the equilibrium rent, so rents are falling even though house prices are rising. For example, when the interest rate decreases from 4 to 2 percent, the aggregate supply of rental property rises increases by 4 percent while the rent falls from 0.22 to 0.21. At the same time, the relative importance of owners of small-sized homes in the rental market increases significantly because with higher house prices and greater leverage, homeowners use rental income to partially cover the interest rate payments on their outstanding mortgage loans. For example, as can be seen in Table 9, when interest rate falls from 4 to 2 percent, both the owners of the large properties as well as the owners of the medium-sized properties account for a smaller percentage of all landlords (54.4 percent to 46.8 percent for large-property owners, and 45.0 percent to 27.9 percent for medium-sized property owners). The opposite trend occurs for owners of small houses, who account for a much larger share of all landlords when the interest rate reaches 2 percent (0.9 to 25.3 percent).

Figure 4 shows that the steady state homeownership rate is constant for interest rates between one and 4 percent. One might expect the homeownership rate to rise as the interest rate decreases from 4 to one percent because a falling risk-free interest rate both decreases the cost of mortgage financing and reduces the attractiveness of saving relative to housing
Table 9: The Distribution of Owned Housing and Landlords Under Different Interest Rates

<table>
<thead>
<tr>
<th>House Size</th>
<th>6% Interest Rate</th>
<th>4% Interest Rate</th>
<th>2% Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% HHs % Landlords</td>
<td>% HHs % Landlords</td>
<td>% HHs % Landlords</td>
</tr>
<tr>
<td>Renter</td>
<td>18.9 0.0</td>
<td>33.7 0.0</td>
<td>34.7 0.0</td>
</tr>
<tr>
<td>Small property</td>
<td>46.2 0.9</td>
<td>13.9 0.9</td>
<td>11.0 25.3</td>
</tr>
<tr>
<td>Medium property</td>
<td>31.4 45.4</td>
<td>46.9 45.0</td>
<td>48.6 27.9</td>
</tr>
<tr>
<td>Large property</td>
<td>3.5 53.7</td>
<td>5.5 54.1</td>
<td>5.7 46.8</td>
</tr>
</tbody>
</table>

investment. However, this is a situation where accounting for equilibrium price effects is critical, as Table 10 illustrates. As can be seen in Column (2), a reduction in $r$ from 4 percent to 2 percent under the restriction that the house price and rent are not allowed to adjust would result in a 15 percent increase in the homeownership rate from 0.66 to 0.81, and an increase in household borrowing because of the lower cost of mortgage financing. At the same time, the fraction of landlords in the economy would rise by 39 percent from 0.10 to 0.49, because households would purchase bigger properties and use rental income to keep up with the mortgage payments. However, in the general equilibrium where the house prices and rents can adjust (Column (3)), the homeownership rate remains constant.⁴⁰ In equilibrium, the decline in the interest rate has no effect on the homeownership rate because the higher house price increases the minimum down payment requirement while a lower $r$ decreases the speed at which the aspiring first-time buyers are able to save up for it. Also, the equilibrium rent decreases slightly from 0.22 to 0.21 even though house prices are rising, which further discourages households from becoming homeowners. All of these effects offset the fact that mortgage interest payments decrease when the interest rate falls.

5.3 Changes in Income

A large body of empirical literature identifies the level and growth rate of income as an important determinant of house price dynamics (see, for example, Poterba (1991), Englund and Ioannides (1997), Muellbauer and Murphy (1997), Malpezzi (1999), and Sutton (2002)). In the United States, real hourly wages increased by 9.4 percent between 1995 and 2005.³¹

Figure 5 summarizes the impact of changes in income on the housing market equilibrium. In our experiment, we assume that household wages rise at the same rate across all wage groups. The model suggests that both house prices and rents increase linearly at about the

³⁰Interestingly, when the interest rate is well above 4 percent, households do not in general borrow to purchase housing properties, preferring to use the accumulated saving to pay for the house upfront. Although households see their savings grow quickly when interest rate is high, they live, on average, in smaller-sized properties, and move up the house size ladder only when sufficient amount of savings was accumulated to pay for the house in cash. At the same time, the supply of rental property contracts significantly, since homeowners substitute rental income for the interest rate income on their savings, and the rent rises. As a result, the price-rent ratio falls below 9.0, increasing the affordability of owning relative to renting. The higher affordability of owning vs. renting in turn further increases the homeownership rate.

³¹This calculation is based on the BLS Current Employment Statistics (CES) real wage data, series ID CES0500000032.
Table 10: The Partial and Equilibrium Effects of the Interest Rate Reduction to 2%

<table>
<thead>
<tr>
<th></th>
<th>Baseline (r = 0.04)</th>
<th>Reduction of Interest Rate to 2%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td><strong>House Price</strong></td>
<td>2.55</td>
<td>2.55</td>
</tr>
<tr>
<td><strong>Rent</strong></td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>Share of Homeowners</strong></td>
<td>0.66</td>
<td>0.81</td>
</tr>
<tr>
<td><strong>Share of Renters</strong></td>
<td>0.34</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>Share of Landlords</strong></td>
<td>0.10</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Share of Owner-Occupiers</strong></td>
<td>0.56</td>
<td>0.32</td>
</tr>
<tr>
<td><strong>Share of Homeowners in Debt</strong></td>
<td>0.64</td>
<td>0.94</td>
</tr>
</tbody>
</table>

same rate as wages. For example, when the wage level increases by 10 percent relative to the benchmark economy, the equilibrium house price and the rent rise by approximately 11 percent. As a result, the house price-rent ratio stays approximately flat. Since the relative price of obtaining housing services through the rental market compared to the market for owned housing remains unchanged, symmetric changes in income of the sort examined here have no effect on the homeownership and landlord rates.

Table 11 helps to explain why the homeownership rate does not rise with income. Again, Column (2) shows the impact of a 10 percent increase in income under the restriction that house prices and rents are not allowed to change (i.e., both house prices and rents are fixed at their equilibrium values from the baseline version of the model). Column (3) reports the impact of a 10 percent increase in income when both house prices and rents are allowed to adjust to clear the housing and rental markets at the higher income level. When house prices and rents are not allowed to adjust, rising income has a substantial impact on the housing market, with the homeownership rate increasing from 66 to 92 percent as more households are able to afford the down payment and mortgage payments required to purchase a house. In addition, many households stop renting out their units as they can more easily cover their mortgage payments: the share of owner-occupied housing increases from 0.56 to 0.71. However, once the house prices and rents are allowed to adjust to higher incomes, homeownership returns approximately to its baseline level (Column (3)). When income increases by 10 percent, the equilibrium house price increases by about the same rate from 2.55 to 2.85, while the equilibrium rent increases from 0.22 to 0.25. The relative cost of renting and owning remains unchanged, keeping the proportions of renters, homeowners, landlords, and owner-occupiers in the economy essentially the same as in the benchmark specification.

The actual changes in the income levels were not, however, symmetric. Heathcote, Perri, and Violante (2009) document the changes in the U.S. earnings inequality between 1967 and 2006. Using the CPS data, the authors find that the real earnings of the bottom decile of the earnings distribution did not, on average, grow between 1985 and 2000, although the earnings of the top earnings distribution grew steadily over the sample period (see their Figure 7). The authors also find that the wage dynamics of the bottom decile of the earnings distribution is very similar to those for the median workers (e.g., workers in the 45-55 precentile of the earnings distribution).
5.4 Combined Effects of Changes in the Market Fundamentals

As discussed in the preceding sections, neither declines in the real interest rate, relaxation of borrowing constraints, nor rising incomes can on their own account for the increase in the price-rent ratio, homeownership rate, and household debt between 1995 and 2005. This section examines the combined effects of changes in these fundamentals on equilibrium housing market outcomes. Figure 6 depicts the percentage deviation of the steady state price-rent ratio from the baseline economy for a range of interest rates and required down payments. Point A represents the calibrated baseline economy with an interest rate on deposits, $r$, of 4 percent and a required down payment, $\theta$, of 20 percent. As the interest rate and the required down payment decrease, the price-rent ratio steadily rises. The price-rent ratio increases by 20 percent over its baseline value when the interest rate is 2 percent and the required down payment is 15 percent. These changes in the interest rate and down payment seem to be a reasonable representation of the recent U.S. experience. For comparison, the U.S. price-rent ratio increased by 36 percent from 1995 to 2005, and by 26 percent between 2000 and 2005.

Table 12 provides a more comprehensive analysis of the simulated effects by showing the percentage deviations in house prices, rents, and the price-rent ratio from their baseline values (Column (1)). To facilitate a comparison of the model’s predictions to the data, Columns (8) and (9) show recent changes in the U.S. data. Columns (2) through (4) show
Table 11: The Partial and Equilibrium Effects of a 10% Increase in Income

<table>
<thead>
<tr>
<th></th>
<th>Baseline (1)</th>
<th>10% Increase in Income Fixed Prices (2)</th>
<th>Equilibrium Prices (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>House Price</td>
<td>2.55</td>
<td>2.55</td>
<td>2.85</td>
</tr>
<tr>
<td>Rent</td>
<td>0.22</td>
<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
<td>Share of Homeowners</td>
<td>0.66</td>
<td>0.92</td>
<td>0.67</td>
</tr>
<tr>
<td>Share of Renters</td>
<td>0.34</td>
<td>0.21</td>
<td>0.33</td>
</tr>
<tr>
<td>Share of Landlords</td>
<td>0.10</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>Share of Owner-Occupiers</td>
<td>0.56</td>
<td>0.71</td>
<td>0.56</td>
</tr>
<tr>
<td>Share of Homeowners in Debt</td>
<td>0.64</td>
<td>0.70</td>
<td>0.64</td>
</tr>
</tbody>
</table>

that when income is held constant, lowering $\theta$ and $r$ raises house prices, lowers rents, and consequently increases the price-rent ratio. Columns (5) through (7) of Table 12 show that increasing wages by 10 percent while decreasing $\theta$ and $r$ does not change the price-rent ratio compared to the scenarios where income is held constant.\(^{33}\) However, the model also predicts that higher income will cause a small increase in rents that is quite close to the growth in rents observed in the United States. As noted above, the actual increase in the house price-rent ratio from 1995 to 2005 was about 36 percent, so a plausible calibration of the model can account for over one-half of the observed increase. Of course, this estimate must be viewed in the context of our admittedly stylized steady state model of the housing market. However, our results suggest that the changes in the interest rate and required down payment observed in the United States had a substantial impact on the price-rent ratio. In addition, the ability of our model to simultaneously predict large increases in house prices and sluggish rents is consistent with recent developments in the U.S. housing market and stands in marked contrast with predictions of simpler models of the housing market which imply that equilibrium house prices and rents must change in the same direction and at the same rate.

In our model, holding house prices and rents constant, when the mortgage interest rate and required down payment fall, the demand curve for rental property shifts inward because households switch from renting to owning as homeownership becomes more affordable.\(^{34}\) At the same time, the supply curve for rental property shifts to the right because when $\theta$ and the interest rate decrease, more households are able to afford down payments and mortgage payments on rental properties. In addition, since both the mortgage rate and rate of return on deposits fall when interest rates decrease, investing in rental property becomes more attractive relative to the alternative of holding bank deposits. The net result of the declining demand and increasing supply in the rental market is a decrease in the equilibrium rent. At the same time, the demand for housing (or homeownership) increases when the interest rate and the required down payment decrease because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the supply of housing

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\(^{33}\)A 10 percent increase in real wages is approximately what was observed in the U.S. between 1995 and 2005.

\(^{34}\)When the downpayment requirement declines from 20 to 10 percent and the interest rate falls from 4 to 2 percent, the homeownership rate increases by 1.4 percent from 0.66 to 0.674.
Table 12: The Combined Effects of Interest Rate, Required Downpayment, and Income Changes

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Changes in $r$ and $\theta$ ($%\Delta$ from Baseline)</th>
<th>U.S. Data ($%\Delta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$r=0.04$ $\theta=0.20$ $r=0.02$ $\theta=0.17$ $r=0.02$ $\theta=0.15$ $r=0.02$ $\theta=0.13$ $\Delta w = 10%$ $\Delta w = 10%$ $\Delta w = 10%$</td>
<td>1995-2005 2000-2005</td>
</tr>
<tr>
<td>house price</td>
<td>2.55</td>
<td>15.7% 16.0% 19.1% 24.7% 28.0% 29.1%</td>
<td>46.3% 31.1%</td>
</tr>
<tr>
<td>rental price</td>
<td>0.225</td>
<td>-3.5% -3.6% -1.3% 4.4% 6.6% 7.5%</td>
<td>7.5% 4.2%</td>
</tr>
<tr>
<td>price-rent ratio</td>
<td>10.8</td>
<td>19.6% 20.0% 20.6% 19.0% 19.6% 20.0%</td>
<td>36.1% 25.9%</td>
</tr>
</tbody>
</table>

Notes: Columns (2)-(7) show percent changes in the equilibrium value of each variable from the baseline model shown in column (1). Columns (8) and (9) show the actual percent changes observed in the U.S. over two different time periods.
is fixed, the equilibrium house price rises. It follows that the price-rent ratio increases as the house price increases and rent falls in response to the change in fundamentals.

6 Conclusion

This paper develops a dynamic equilibrium model of the housing market in which both house prices and rents are determined endogeneously. We use the model to study the relationship between the steady state house price-rent ratio and fundamentals such as the interest rate, required down payment, and income. This analysis is motivated by the fact that although the price-rent ratio is a widely used economic indicator, its determinants are not well understood. Without a theoretical understanding of how the price-rent ratio is determined, it is not possible to determine whether observed changes in the relationship between house prices and rents reflect changing fundamentals or an asset price bubble.

The model predicts that the combination of low interest rates, reduced down payment requirements, and rising wages observed in the United States leads to a large increase in the steady state, rational expectations equilibrium price-rent ratio. However, changes in these fundamentals are capable of explaining only about one-half of the 36 percent increase in the
price-rent ratio observed between 1995 and 2005. At the same time, changes in fundamentals generate increases in the homeownership rate and household debt that are consistent with the recent U.S. experience.

7 Appendix A

7.1 Finding Equilibrium in the Housing and Rental Markets

Equilibrium in the housing and rental markets is formally defined by the conditions presented in Section 3. In practice, the market clearing rent \((\rho^*)\) and house price \((q^*)\) are found by finding the \((q^*, \rho^*)\) pair that simultaneously clear both the housing and shelter markets in a simulated economy. The market clearing conditions for a simulated cross section of \(N\) agents are

\[
\sum_{i=1}^{N} h_i'(q^*, \rho^*|x) = H \quad (13)
\]

\[
\sum_{i=1}^{N} s_i'(q^*, \rho^*|x) = H. \quad (14)
\]

The optimal housing and shelter demands for each agent are functions of the market clearing steady state prices and the agents other state variables \((x)\). Solving for the equilibrium of the housing market is a time consuming process because it involves repeatedly re-solving the optimization problem at potential equilibrium prices and simulating data to check for market clearing until the equilibrium prices are found. The algorithm outlined in the following section exploits theoretical properties of the model such as downward sloping demand when searching for market clearing prices. Taking advantage of these properties decreases the amount of time required to find the equilibrium far below that of a more naive search algorithm.

7.2 The Algorithm

Let \(q_k\) represent the \(k^{th}\) guess of the market clearing house price, let \(\rho_k\) represent a guess of the equilibrium rent, and let \(\rho_k(q_k)\) represent the rent that clears the market for housing conditional on house price \(q_k\). The algorithm that searches for equilibrium is based on the following excess demand functions

\[
ED^h_k(q_k, \rho_k) = \sum_{i=1}^{N} h_i'(q_k, \rho_k|x) - H \quad (15)
\]

\[
ED^s_k(q_k, \rho_k) = \sum_{i=1}^{N} s_i'(q_k, \rho_k|x) - H. \quad (16)
\]
The equilibrium prices $q^*$ and $\rho^*$ simultaneously clear the markets for housing and shelter, so

$$ED_h^k(q^*, \rho^*) = 0 \quad (17)$$

$$ED_s^k(q^*, \rho^*) = 0. \quad (18)$$

The following algorithm is used to find the market clearing house price and rent.

1. Make an initial guess of the market clearing house price $q_k$.

2. Search for the rent $\rho_k(q_k)$ which clears the market for owned housing conditional on the current guess of the equilibrium house price, $q_k$. The problem is to find the value of $\rho_k(q_k)$ such that $ED_h^k(q_k, \rho_k(q_k)) = 0$. This step of the algorithm requires re-solving the agents’ optimization problem at each trial value of $\rho_k(q_k)$, simulating data using the policy functions, and checking for market clearing in the simulated data. One useful property of the excess demand function $ED_h^k(q_k, \rho_k(q_k))$ is that conditional on $q_k$, it is a strictly decreasing function of $\rho_k$. Based on this property, $\rho_k(q_k)$ can be found efficiently using bisection.

3. Given that the housing market clears at prices $(q_k, \rho_k(q_k))$, check if this pair of prices also clears the market for shelter by evaluating $ED_s^k(q_k, \rho_k(q_k))$.

   (a) If $ED_s^k(q_k, \rho_k(q_k)) < 0$ and $k = 1$, the initial guess $q_1$ is too high, so set $q_{k+1} = q_k - \varepsilon$ and go to step (2). This initial house price guess $q_1$ is too high if $ED_s^k(q_k, \rho_k(q_k)) < 0$ because $ED_s^k(q_k, \rho_k(q_k))$ is decreasing in $q_k$.

   (b) If $ED_s^k(q_k, \rho_k(q_k)) > 0$ set $k = k + 1$ and $q_{k+1} = q_k + \varepsilon$ and go to step (2).

   (c) If $ED_s^k(q_k, \rho_k(q_k)) = 0$, the equilibrium prices are $q^* = q_k$, $\rho^* = \rho_k(q_k)$, so stop.

8  Appendix B

8.1  Survey of Consumer Finances 2007

[To be added.]
References


