Place-Based Redistribution in Simple Location-Choice Models*

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Abstract

We study the planning problem of a standard location-choice model to discuss why a planner might optimally redistribute output across locations. In this model, ex-ante identical households value consumption and housing and choose a location in which to live, where locations vary in productivity, amenities, and the quantity of housing. Two features of this model lead the planner to redistribute output. First, the supply of housing is fixed in each location and subject to a congestion externality. Second, households randomly and unobservably vary in their "attachment" to any given location, affecting both household location choice and utility. We demonstrate that researcher choice in modeling attachment critically affects the size and direction of optimal transfers of output across locations. This is a key problem as the location-choice data do not discipline this facet of the model: A plethora of frameworks may be observationally equivalent, but predicted optimal transfers can vary greatly across approaches. We propose a simple adjustment to the planning problem that removes the influence of the attachment-modeling choice on predicted optimal policy but preserves the planner's incentives to redistribute across locations in response to housing congestion and/or other externalities.

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1. Introduction

For decades, federal, state and local governments have directly or indirectly redistributed income across locations. This redistribution can take many forms: It can be a subsidy for development of new low-income housing (Davis et al., 2019); a subsidy to local businesses operating in low-income areas such as Empowerment Zones (Busso et al., 2013); a large-scale government works projects (Kline and Moretti, 2014); or other forms. Thus, a central area of investigation in Economics is to understand the context in which redistribution across locations improves welfare. Recent papers by Fajgelbaum and Gaubert (2019), Rossi-Hansberg et al. (2020) and Gaubert et al. (2020) extend this tradition by studying optimal transfers of income across households and locations using modern, sophisticated equilibrium location-choice models. The models of these authors include well-documented externalities in production and two types of households, low- and high-skill. The goal of the authors is to quantify, using the filter of the calibrated model, transfers across people and locations that improve expected utility for reasons of both efficiency and equity.

In this paper, we take a step back and study optimal place-based redistribution in a simple location-choice model that in many ways is a canonical model of Urban Economics. To be clear, when we say "place-based redistribution," we are referencing an environment in which a planner optimally transfers output across locations, such that consumption of households in a given location is not equal to output produced in that location. We want to understand the basic properties and mechanisms underlying optimal place-based redistribution arising from a simple model that is familiar to many Economists, before adding complexity and more realism. Perhaps our most surprising result is that optimal place-based redistribution is a robust feature of simple location-choice models – models that do not include production externalities, such as agglomeration, or any desire to redistribute across different types of people (i.e. high- and low-skill workers) that sort into different locations. Consequently, we believe discussion of optimal redistribution in more realistic and complicated location-choice environments should start with the redistribution we discuss in our simple model as a "baseline." Any additional redistribution arising from a more complicated model can then be understood as a consequence of the additional complexity.

In the model we study, ex-ante homogeneous households choose where to live, with the natural restriction that they must live and work in the same location. In each location, households receive utility from consumption and housing subject to a draw of a location-specific attachment variable that varies across locations and households. We show that in this model, a planner will want to transfer income across locations for two reasons. First, housing is each location is in fixed supply, and households must live and work in the same location. Second, the location-specific attachment draws are not insured. These draws not only determine optimal location choice, but also the level utility and the marginal utility of consumption. To increase average (expected) utility, the planner will redistribute income across locations given how households sort themselves into locations, the level of utility, and the marginal utility of consumption, all of which are partially based on the draws.

We are troubled by this last result, because we argue researchers have essentially no guidance from the data as to how to model these draws, and this key modeling choice influences predicted optimal transfers. Restated, researchers can model these draws in multiple ways such that the full set of testable predictions of the location-choice model are indistinguishable. To illustrate this point, we compare the planning solution of the simple location-choice model with only two locations, but when the location-specific draws are from (a) the Fréchet distribution and then (b) a closely related distribution, the Weibull. We pick the parameters of both distributions to match the model's predicted elasticity of relative population with respect to relative wages, a commonly chosen calibration target. The Fréchet and the Weibull generate the exact same distribution of location choices: There is nothing to tell them apart. But the choice of the researcher as to which of these two distributions to use determines the solution to the planner's problem and the size and direction of optimal place-based distribution. With the Fréchet distribution, a planner will always want to transfer income from high- to low-income locations. With the Weibull, the planner may want to transfer income from the high- to low-income location or from the low- to the high-income location. Regardless of the direction of the transfer, the size of the transfer is different than when the preference draws are from the Fréchet.

We conclude the paper by proposing an adjustment to the planner's problem where the planner maximize *weighted*-average utility. We specify the weights to "undo" the impact of assumptions of the distribution of the preference draws on optimal redistribution across locations; our proposed adjustment will have this effect for almost any standard location-choice model and any distribution of the preference draws. We study the properties of our proposed adjusted planning problem for a two-location version of the location-choice model of the paper, and, an amended version in which one location has a small agglomeration externality. In both environments, we derive the planner's solution for optimal redistribution of income across locations when households are assumed to not draw location-specific preferences. We simulate the model and compute the solution to the regular and the adjusted planner's problem when location-specific preferences are drawn in these environments from the Fréchet distribution and the Weibull distribution. In the unadjusted planner's problem, the direction of the transfers across locations varies between the Fréchet and the Weibull settings, and neither delivers the analytic solutions we derive in the absence of location-preference draws. In the adjusted planner's problem, optimal transfers in the Fréchet and Weibull settings are identical, and equal to the optimal transfers we compute analytically when the models do not have location-preference draws.

2. Planning Problem, Full Model

We start by specifying a simple, location-choice model in which the economy consists of a measure 1 of ex-ante identical households. Throughout the paper, we sometimes refer to this as a canonical model of Urban Economics. In this model, each household must choose where to live from one of $n = 1, \dots, N$ discrete locations. Households value consumption, which is produced and transferrable across locations, and housing, which is not produceable and not transferrable across locations. Each household living in location n produces z_n units of output; and each household in location n lives in a house of size h_n which is equal to H_n/L_n, where H_n is the stock of housing and L_n is the measure of households living and working in n.

Denote c_n as consumption enjoyed by each household living in location n, not necessarily equal to z_n. Utility of household i choosing to live in location n is:

$$u_{in} = A_n c_n^{1-\alpha} h_n^{\alpha} e_{in}$$

A_n are amenities freely enjoyed by all households living in location n. e_{in} is a level of attachment to location n by household i that varies across locations and households. Each household draws and observes e_{in} for $n = 1, \ldots, N$ before making a location choice and ex-ante identical households differ only with respect to these draws. In the rest of this section we assume (as is common) that the e_{in} are drawn iid across locations and households from the Fréchet distribution with shape parameter ν.

Consider a planner with the objective to maximize expected utility subject to satisfying aggregate feasibility, $\sum_n z_n L_n = \sum_n c_n L_n$, population feasibility, $1 = \sum_n L_n$, and respects that households choose the location offering the maximum value of u_{in}, i.e. household i chooses n^* when $n^* = \text{argmax}\,\{u_{in}\}_{n=1}^{N}$. Define u_n as utility from location n prior to the realization of the e_{in}, that is $u_n \equiv u_{in}/e_{in}$. Given the assumed distribution of e_{in}, the probability a household chooses location n, L_n, is equal to $(u_n/U)^{\nu}$, where $U = \left(\sum_n u_n^{\nu}\right)^{1/\nu}$. U is proportional to expected utility.

Given any desired, exogenously given, (federal) government spending of G, we show in the appendix that a planner that maximizes U will set c_n as follows

$$c_n \;=\; (1-\tau)\, z_n \;+\; T \tag{1}$$

where

$$\tau \;=\; \left(\frac{1+\alpha\nu}{1+\nu}\right) \quad \text{and} \quad T \;=\; \tau \sum_n z_n L_n \;-\; G \tag{2}$$

In other words, the planner takes a constant percentage τ of labor income in each location, and rebates all proceeds net of government expenditures as a lump sum rebate.[1] At a typical calibration of $\alpha = 0.25$ (Davis and Ortalo-Magné, 2011) and $\nu = 2$ (Rossi-Hansberg et al., 2020), $\tau = 0.5$: The planner takes 50 percent of labor income from each location, independent of amenities or the aggregate quantity of housing in that location.

[1] It can be shown the planner's solution is identical when household utility is of the form $u_{in} = \nu \ln A_n + \nu\,(1-\alpha) \ln c_n + \nu\alpha \ln h_n + \epsilon_{in}$ and ϵ_{in} is drawn from the Type 1 Extreme Value distribution.

3. Housing Only, $\alpha > 0$ and $\nu \to \infty$

To understand this result, we start by considering an even simpler model with no location-specific preference draws, i.e. where $e_{in} = 1$ for all n and i. In this environment, the planner will always want to transfer some output from high-productivity to low-productivity places and the amount of redistribution will depend on the preference for housing relative to consumption. To see this, consider the limiting case of $\nu \to \infty$ such that $\tau = \alpha$ in equation (2). The planner will transfer output between any two locations n and m such that

$$c_n - c_m = (1-\alpha)(z_n - z_m)$$

If $z_m > z_n$, the net consumption subsidy to residents of location n from residents of m is

$$(c_n - c_m) - (z_n - z_m) = \alpha (z_m - z_n)$$

We have been asked to discuss the degree to which taxes and transfers are required in the competitive equilibrium to achieve the planner's desired allocation. The answer, it turns out, depends on how rental income from housing is distributed in the population.[2] Denote I_n as per-household income in location n in the competitive equilibrium without taxes and transfers. Given households have Cobb-Douglas preferences for consumption and housing, they choose c_n and h_n to satisfy

$$c_n = (1-\alpha) I_n \quad \text{and} \quad r_n h_n = \alpha I_n \tag{3}$$

where r_n is the rental price per unit of housing in n. Now suppose that I_n includes labor income z_n and, critically, a lump-sum redistribution of economy-wide rental expenditures,

[2] To our knowledge, Eeckhout and Guner (2017) are the first to observe the importance of housing institutions in determining optimal taxation in a location-choice model. For recent discussions of the unequal geographic burden of federal taxation and redistribution across locations see Albouy (2009) and Albouy (2012).

i.e. $I_n = z_n + T$ where $T = \sum_n (r_n h_n) L_n$.[3] This implies

$$T = \alpha \sum_n I_n L_n = \alpha \sum_n (z_n + T) L_n = \left(\frac{\alpha}{1-\alpha}\right) \sum_n z_n L_n$$

implying

$$c_n = (1-\alpha) z_n + \alpha \sum_n z_n L_n$$

When economy-wide housing rents are lump-sum rebated, the competitive equilibrium without government taxes or transfers yields the *identical* allocation as the solution to planner's problem when $G = 0$.[4] If $G > 0$, the planning solution is achieved when the government collects all aggregate rental income and then lump-sum rebates any taxes collected remaining after G is taken out.[5]

What happens when housing rents are not assumed to be lump-sum rebated in the competitive equilibrium? Consider an extreme case in which households in city n only receive, lump-sum, housing rents paid by residents of city n, i.e.

$$I_n = z_n + T'_n \quad \text{where} \quad T'_n = r_n h_n$$

This is similar to a society in which all households are owner-occupiers and none have a mortgage.[6] The competitive equilibrium without taxes and transfers will have as an allocation $c_n = z_n$.[7] Without taxes and transfers, the consumption differential between any two cities n and m in this environment is equal to $z_n - z_m$. A government that wants to implement the planning solution must set tax rates equal to α percent of labor income and

[3] A common assumption to generate this lump-sum redistribution is that each household owns an equal share of a REIT that owns all of the housing stock in the economy.

[4] The assumption of Cobb-Douglas preferences for consumption and housing is not key to this result. We can show that lump-sum redistribution of housing rents is sufficient given any constant-returns utility function; this result is available on request.

[5] In the event that G is larger than aggregate housing rents, the government must levy a head tax.

[6] A natural research question investigates how actual home-ownership affects both the planning solution and required taxes and transfers. Allowing for home-ownership and associated complications is beyond the scope of this paper as well as that of nearly all static location-choice models.

[7] It is quickly shown that $r_n h_n = [\alpha / (1-\alpha)] z_n$ and thus $c_n = (1-\alpha) I_n = z_n$.

then rebate the proceeds lump sum.

Rental income is central to redistribution in the planning solution because housing is supplied inelastically and households must work where they live. Consider the implications of moving ΔL more people into location n. Given $h_n = H_n/L_n$ and H_n is fixed, an increase of ΔL residents to location n decreases the available housing to all existing residents of location n. Aggregate utility of existing residents in that location declines by $\alpha u_n \Delta L_n$. To keep utility constant, the planner must increase total consumption allocated to in city n by $\alpha (1-\alpha)^{-1} c_n \Delta L_n$ units. From equation (3) this is equal to $r_n h_n \Delta L_n$. In other words, when ΔL_n people move to location n, the dollar amount of the loss of utility of existing residents in location n is equal to the housing rents paid by the new movers. In conclusion, the planner takes this amount from the new residents such that they internalize the cost they impose on existing residents. The planner "taxes" all households due to the fact that each household reduces housing available to everyone else; the tax rate is the same in each location because rental expenditures are a fixed fraction of consumption in each location; and aggregate taxes collected by the planner (for redistribution or use in government spending) is equal to aggregate rental expenditures.

The planner takes τ from each worker to correct the congestion externality imposed by any individual resident on the population. This delivers the optimal allocation of people to locations. The planner redistributes the proceeds lump-sum so as not to distort the location decision.

4. Shocks Only, $\alpha = 0$ and $0 < \nu < \infty$

Now that we have studied the role of housing in determining place-based redistribution, we now investigate how location-attachment draws affect the planner's solution in a location-choice model without housing. Jumping to the end, we show that how researchers choose to model these draws can radically alter predicted optimal place-based redistribution. We find this result quite troubling because fundamentally the attachment draws are not observable. Many different modeling choices can yield literally the identical likelihood function over a location-choice data set (implying, for example, identical distributions of the population across locations, identical population elasticities with respect to wages, etc.). Each modeling choice is likely to yield a different prediction for optimal place-based redistribution.

Here is a very simple, illustrative example. Consider a model in which there are only two locations, n and m; utility in location n is $\ln(c_n)$ and utility in location m is $\ln(c_m + e_{im})$; and, e_{im} is an attachment draw for household i that is realized before the location decision is made and only applies when the household lives in location m. The presence of e_{im} generates a population elasticity with respect to relative wages. For a household to live in location m, it must be the case that $e_{im} > c_n - c_m$. This implies the marginal utility of consumption of every household choosing to live in location m, $1/(c_m + e_{im})$, is less than the marginal utility of any household choosing to live in location n, $1/c_n$. A planner maximizing average utility has incentives to transfer consumption from location m to location n to exploit differences in the marginal utility of consumption, regardless of the relative productivity of either location. However, if researchers assume the attachment draw is in location n instead of m, such that utility in locations n and m are $\ln(c_n + e_{in})$ and $\ln(c_m)$, using similar reasoning we can show the planner will always want to transfer resources from n to m. Ultimately, the researcher's choice of whether to include e_{im} or e_{in} in the model determines if the planning solution involves transfers from m to n or from n to m.

This stark example shows the importance of the attachment draws – how they enter utility and the relative variance of the draws across locations – in determining optimal transfers across locations. The functional form of the attachment draws matters as well. Returning to the simple example of section 2, without housing (i.e. $\alpha = 0$) utility in city n is $A_n c_n e_{in}$ where e_{in} are assumed to be drawn iid across households and locations from the Fréchet distribution. The planner's optimal "tax rate" on income, τ in equation (2), is equal to $1/(1+\nu)$, and this tax rate does not vary with productivity z_n or amenities A_n. The planner always transfers resources from places with high productivity to places with lower productivity, such that across-location variation in consumption is less than across-location variation in income, i.e. $c_n - c_m = (1-\tau)(z_n - z_m)$.

This stark result, while convenient, does not hold if we assume the e_{in} are drawn from a different, but very similar distribution. Consider the case where the location preference shocks are drawn from the closely-related Weibull distribution with shape parameter k.[8] Figure 1 graphs the pdfs of these two distributions, with the values $\nu = 2$ for the Fréchet (red solid line) and $k = 2$ for the Weibull (blue dashed line) chosen for reasons we discuss later. These two distributions look similar but the thickness of the right tails are different, and in the case of predicting optimal transfers this might matter quite a bit: Households only move to low-wage locations if their location-preference draw is sufficiently large.

It is possible to show that when the e_{in} terms are drawn from the Fréchet with parameter ν, the elasticity of relative population with respect to relative consumption is equal to ν, i.e.

$$\frac{d\ln\left(L_n/L_m\right)}{d\ln\left(c_n/c_m\right)} = \nu$$

Rossi-Hansberg et al. (2020) suggest this elasticity is approximately 2. We show in Appendix B for an economy in which there are only two locations that when e_{in} terms are drawn from the Weibull, this elasticity of relative population with respect to relative consumption is equal to k, the shape parameter of the Weibull distribution. For this reason, we set $k = \nu = 2$ in the examples that follow.

We use simulations to compute optimal transfers across locations when the e_{in} terms are drawn from the Weibull. In figure 2 we show optimal transfers across locations in a two-location economy when the e_{in} terms are drawn from the Fréchet distribution at $\nu = 2$ and the Weibull distribution at $k = 2$. The x-axis of this figure is the ratio of productivity in location n to location m – we consider values in the range $0.8 \leq z_n/z_m \leq 1.0$ – and the y-axis is the ratio of consumption in the two locations after the planner has optimally redistributed income. The solid-black line traces out points where the ratio of consumption is equal to the ratio of income and no redistribution has occurred. Any value of c_n/c_m that lies above the solid-black line indicates that the planner is redistributing income from location m, the more productive location, to location n. Conversely, any value of c_n/c_m that lies below the solid-black line indicates the planner redistributes income from the less-productive to the more-productive location.

[8] For reference, the pdf of the Fréchet with parameter ν is $\nu x^{-1-\nu}e^{-x^{-\nu}}$ and the pdf of the Weibull with parameter k is $kx^{k-1}e^{-x^k}$.

The solid red line in this graph shows the choice of the planner for c_n/c_m given z_n/z_m when the e_{in} terms are drawn from the Fréchet. This solid red line is everywhere above the 45 degree line, consistent with the redistribution from high- to low-income locations prescribed by section 2. As equations (1) and (2) show, the planner's choice of c_n/c_m when the shocks are drawn from the Fréchet only depends on z_n and z_m and does not depend on the values of fixed amenities in locations n or m, A_n and A_m.

The blue lines on this graph show similar results when the e_{in} terms are drawn from the Weibull distribution. We consider three values for A_m/A_n: 1.0 (dashed line), 1.5 (dash-dot line), and 3.0 (dash-dash-dot-dot line). The results for $A_m/A_n = 1$ highlight how the differences in the distributions affect expected average marginal utilities, as the decision to locate in location 1 satisfies $e_{in} > (c_m/c_n)\, e_{im}$ and the marginal utilities in each location are $A_n e_{in}$ and $A_m e_{im}$, respectively. The planner still redistributes from the more productive to less productive location, as the blue-dashed line lies above the solid black (no-redistribution) line, but the planner redistributes less than when shocks are drawn from the Fréchet, as the blue-dashed line lies everywhere beneath the red line. That said, when the e_{in} terms are drawn from the Weibull the value of A_m/A_n also determines the direction of transfers. For both A_m/A_n equal to 1.5 and 3.0, the planner chooses to transfer resources *from* residents of the low-productivity location and *to* residents of the high-productivity location for all values of z_n/z_m we consider.

In our view, these results are potentially problematic for researchers studying place-based redistribution. We do not know the distribution of the unobserved attachment draws (or even how they should enter utility), but these features determine not only the size of transfers across locations but also the *direction*. In the example, we calibrated the parameter of the Fréchet and the Weibull distribution to match a typical moment used in calibrations of models of Urban Economics, the sensitivity of the population to changes in relative wages. Yet, even with this discipline, transfers with the Weibull shocks in our simple model had the potential to be completely different than transfers with the Fréchet, depending on the values of the relative amenities, A_m and A_n.

Why doesn't the elasticity of location choice with respect to relative wages pin down transfers? This elasticity is informative about the distribution of preference draws for people that are only marginally attached to their location. The optimal amount of redistribution across locations depends on the average marginal utility of consumption of households in each location. These average marginal utilities depend on the preference draws for households that are unlikely to move. For example, if two distributions have different shapes for these households – i.e. if the right tails of the distributions in figure 1 are very different – then optimal transfers are likely to be different as well. In principle, the shape of these distributions is simply not identifiable, as many different distributions will generate the exact same location choices.

5. Adjusting the Planning Problem

So far, we have discussed two reasons why the planner would want to redistribute resources across cities: (1) to account for the fact that location choices of individual households affect the quantity of housing available to all others and (2) to account for the fact that location-attachment draws that determine location choice are uninsured and affect location choices, the level of utility, and marginal utility. As mentioned, we find this second reason troublesome since researchers do not observe the distribution of these attachment draws.[9] In this section we propose an adjustment to the planning problem that preserves what we believe are fundamental economic reasons for transfers across locations, for example those related to housing congestion or externalities, but eliminates transfers that arise due to the presence of the attachment draws.

Define $\mathcal{O}$ as the planner's objective function. We propose

$$\mathcal{O} = \int_i \omega_i u\left(c_n^i, h_n^i, e_n^i\right) di \tag{4}$$

This is weighted-average utility of all households in the economy after all location preference shocks have been revealed and all location-decision decisions have been made. To eliminate transfers that arise only due to the distribution of the e_n^i, the weight for each household, ω_i, should be equal to the inverse of the marginal utility of consumption for that household. The

[9]Additionally, we are unsure how to use data to guide how these draws should affect utility.

ω_i can be determined with the following straightforward, iterative computational procedure:

Step 1. Draw all values of e_{in} for all households all i and locations n. These are held fixed in all model simulations. Set $\omega_i = 1$ for all households.

Step 2. Solve the planning problem to determine optimal transfers across locations when the planner maximizes the objective specified in equation (4).

Step 3. Record each household's location decision and marginal utility of consumption after the optimal policy of step 2 is implemented. Denote n_i^* as the chosen location for household i and $\mu_i(n_i^*)$ as the marginal utility of consumption for household i at n_i^*.

Step 4. Redefine the weight to be used in equation (4) for each household as the inverse of the marginal utility of consumption, $\omega_i \equiv [\mu_i(n_i^*)]^{-1}$.

Step 5. Repeat Steps 2-4 until the set of predicted optimal transfers across locations converges.

Why does this work? Due to the way in which we define ω_i, a small change in consumption to any household will affect $\mathcal{O}$ by exactly the same amount. This implies the value of the planner's objective will not increase simply by shuffling consumption around (if that were feasible) due to differences in the marginal utility of consumption resulting from different draws of e_{in}. For this reason, once the weights are applied the planner only has incentives to redistribute based on housing congestion or other externalities.

To illustrate how this might work in practice, consider the canonical model described in section 2 with e_{in} drawn iid from the Fréchet and from the Weibull. Assume, as before, $\nu = k = 2$ and consider the specific example setting $A_m = 1.5$, $A_n = 1.0$, $z_m = 1.1$, $z_n = 1.0$ and $\alpha = 0.25$. n in this case is a low-productivity, low-amenity location. The 4 panels of Figure 3 show predictions for optimal place-based redistribution. In each panel, the value of the planner's objective function is on the y-axis and the x-axis shows the size and direction of the net subsidy from m to n, where we define the net subsidy (as before) as $(c_n - c_m) - (z_n - z_m)$. If the net subsidy is positive, the planner transfers income from the high- to the low-productivity location, and if it is negative the planner transfers income in the opposite direction. We know in the case of the canonical model with no location preference draws, i.e. $e_{in} = 1$ always, that the net subsidy is $\alpha\,(z_m - z_n)$ which is equal to 0.025 given our parameterization. The influence of the assumed distribution of the location-preference draws on results can be interpreted as the difference between the computed optimal transfer and 0.025.

The left two panels shows the optimal transfers when the planner maximizes unweighted expected utility, i.e. when $\omega_i = 1$ for all i. Given z_n and z_m have the interpretation of being the wage rates in locations n and m, when the location-preference draws are Fréchet, top-left panel, the planner chooses to transfer resources from the high- to low- wage location: The net subsidy to n is 0.05. The bottom-left panel shows that when the draws are Weibull, the planner chosen to transfer resources from the low- to high-wage location: The net subsidy to n is -0.024. Thus, the distribution of e_{in} determines both th size and the direction of optimal transfers. The right two panels show optimal transfers of the corrected planning problem in equation (4) after the weights ω_i have converged. For both the Fréchet (top-right) and Weibull (bottom-right) draws, the planner redistributes from the high- to the low-productivity city and the net subsidy is 0.025, exactly the analytic solution to the model lacking location-preference draws.

Next, we add add agglomeration effects to the model to show how our proposed correction affects predicted optimal redistribution in the event of a production externality. Define output in each city, y_n, as

$$\begin{array}{lcccccc} \text{City n, no change} & y_n & = & w_n L_n & w_n & = & z_n \\ \text{City m, agglomeration} & y_m & = & w_m L_m & w_m & = & z_m L_m^{\delta} \end{array}$$

where $\delta > 0$ is the agglomeration effect. We do not change any other facet of the model. This model has the virtue of being simple enough such that we analytically derive in Appendix C the solution to the planner problem for optimal consumption in each location in the absence of location-preference draws, i.e. when $e_{in} = 1$ and $e_{im} = 1$ for all households i. This is:

$$\begin{aligned} c_n &= (1-\alpha)\ \left[w_n - \delta y_m\right] + \alpha \cdot GDP \\ c_m &= (1-\alpha)\ \left[(1+\delta)\, w_m - \delta y_m\right] + \alpha \cdot GDP \end{aligned}$$

which implies a net subsidy of

$$(c_n - c_m) - (w_n - w_m) \quad = \quad \alpha\,(w_m - w_n) - (1-\alpha)\,\delta w_m$$

Although the exact value of the net subsidy will depend on L_m, given our parameterization it will be very nearly equal to 0.

Figure 4 shows the impact of the correction on the planner's problem in this model. We set $\delta = 0.03$ but otherwise all parameters and all values of A and z terms are the same as in the previous example. Figure 4 follows the same format as figure 3: The top two panels show results for the Fréchet draws and the bottom two panels show results for the Weibull draws; the left two panels show results when the planner maximizes expected utility and the right two panels show results when the planner maximizes equation (4).

The results when the planner maximizes expected utility are similar to those when there is no externality: In the case of Fréchet location shocks, top-left panel, the planner transfers resources from the more productive location, location m, to the less productive location, location n, and in the case of the Weibull location draws, bottom-left panel, the transfers go in the opposite direction. In contrast, once the planner solves the corrected problem, the planner chooses not to transfer any resources from location m to location n – exactly the same as the analytic solution we derived when $e_{in} = e_{im} = 1$ for all households.

6. Conclusion

Our investigation of optimal place-based redistribution in simple location-choice model yields three main insights. First, the presence of a fixed stock of housing in eah location is sufficient to generate optimal transfers across locations. The planning allocation and the allocation arising from a competitive equilibrium only coincide when aggregate housing rents are lump-sum rebated; otherwise taxes and transfers are needed. Second, the presence of uninsured location-preference draws also generates desired redistribution across locations. The specific distribution of the draws (as well as the way the draws are assumed to enter utility) affect the size and direction of predicted optimal transfers. As we emphasize, we find this last result troublesome, since data do not provide guidance on these preferences. We therefore propose an adjustment to the planning problem such that a planner's predicted optimal redistribution of income across locations does not depend on the way in which unobserved location-preference draws are modeled.

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Figure 1: Pdfs of Fréchet ($\nu = 2$) and Weibull ($k = 2$)

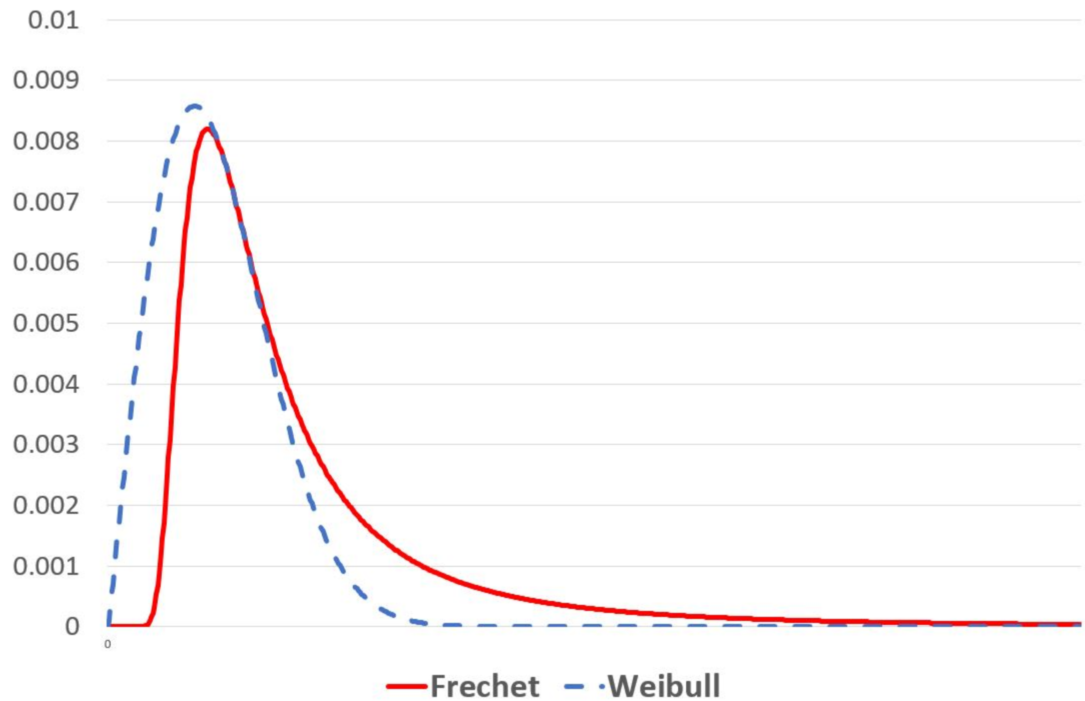


Figure 2: Optimal Transfers in a Simple Urban Model, $\nu = k = 2$

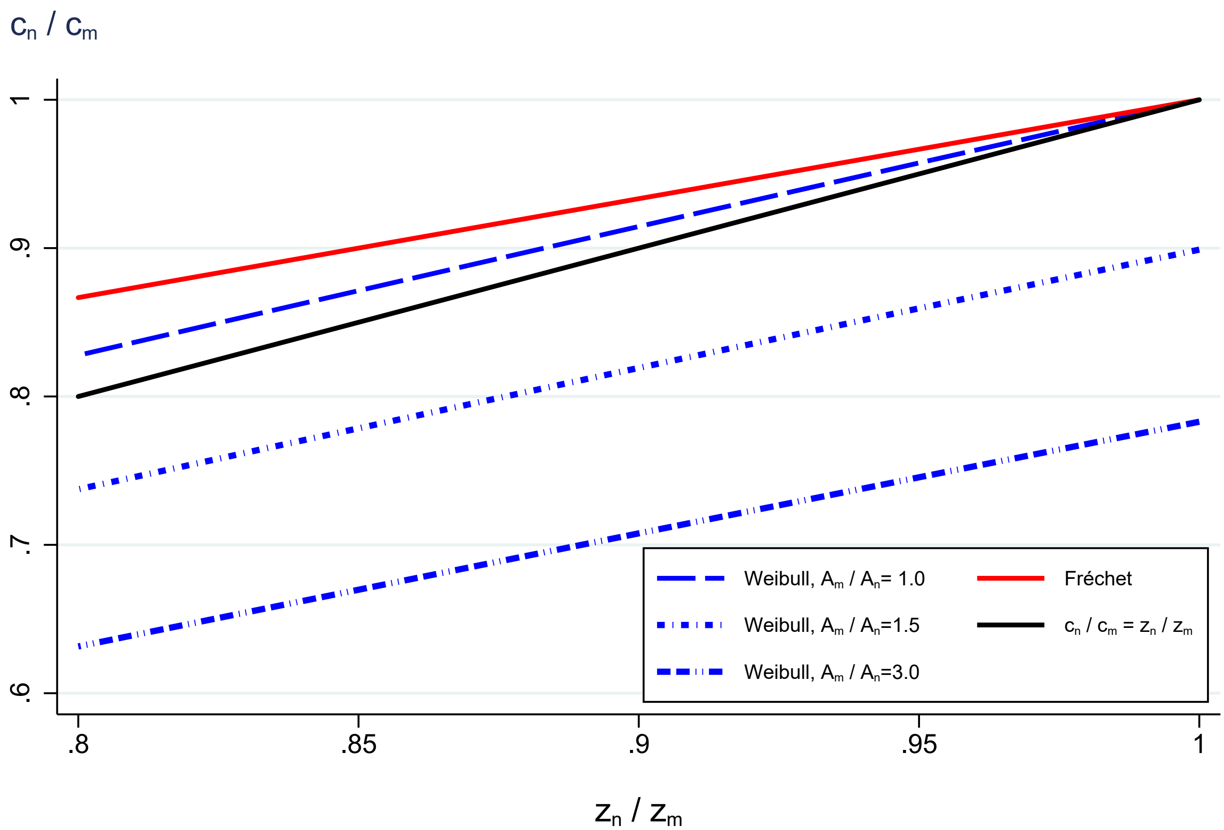
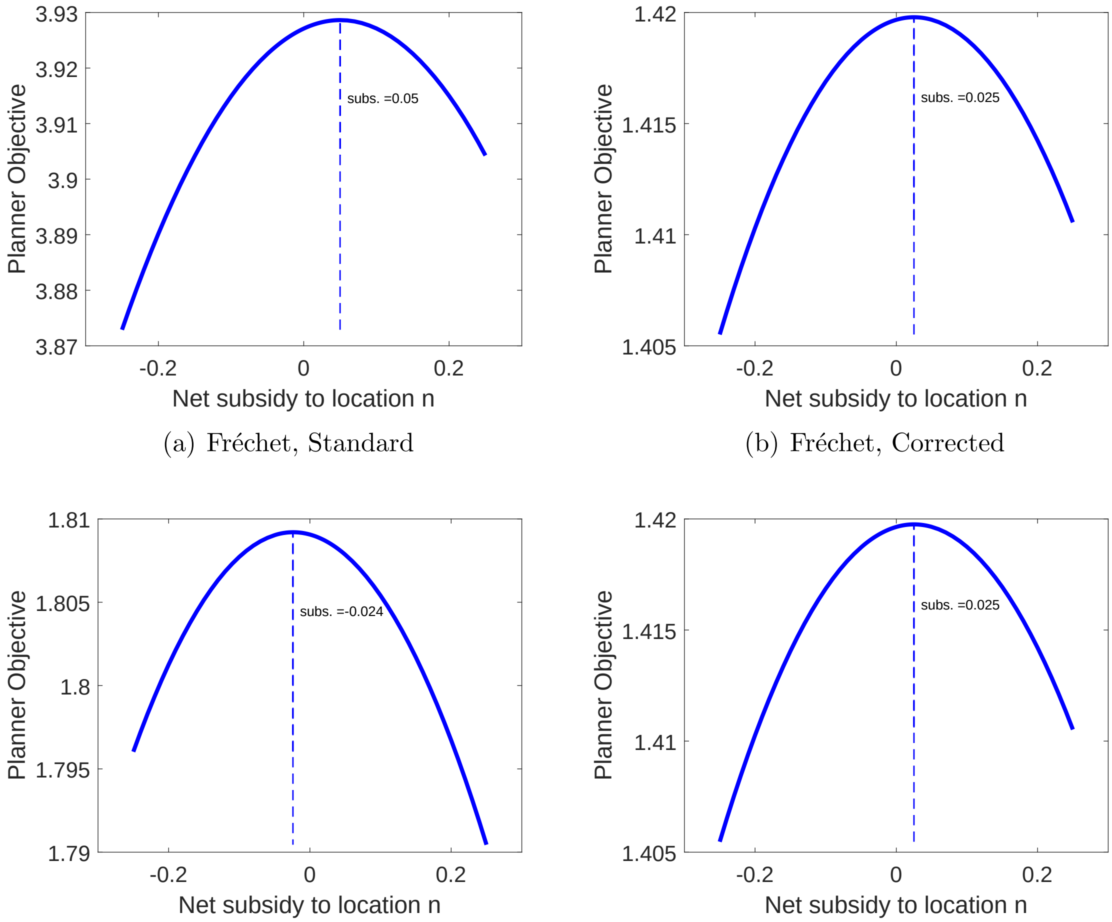


Figure 3: The Impact of the Correction to the Planner's Problem
Model with No Agglomeration Externality



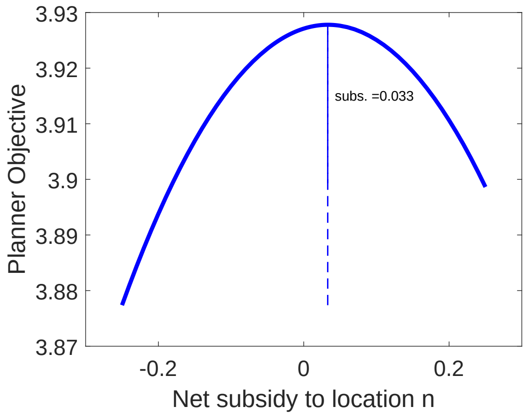
(a) Fréchet, Standard

(b) Fréchet, Corrected

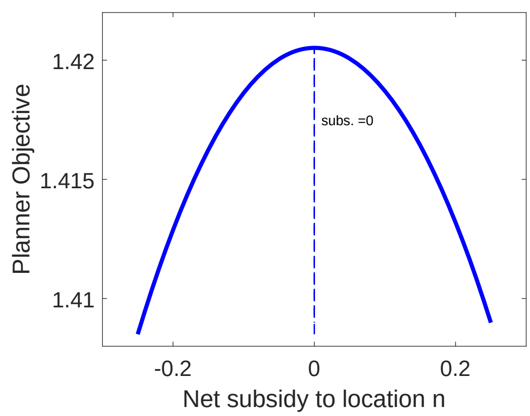
(c) Weibull, Standard

(d) Weibull, Corrected

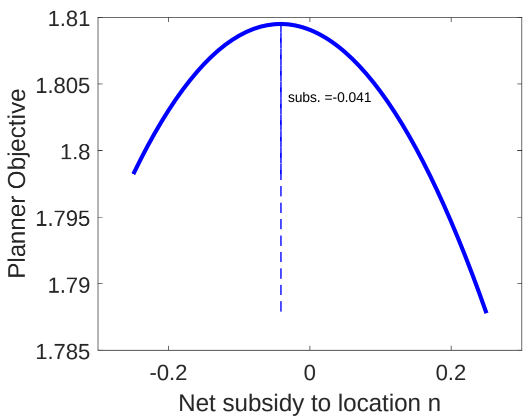
Figure 4: The Impact of the Correction on the Planner's Problem
Model with Agglomeration Externality



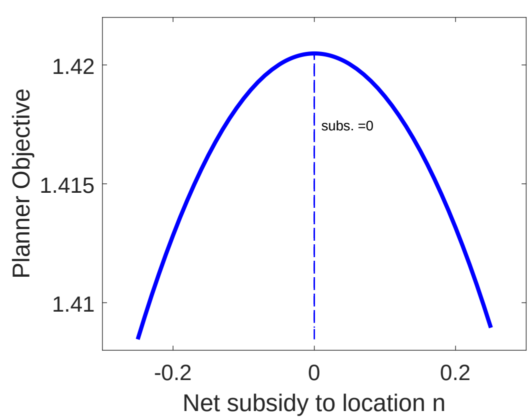
(a) Fréchet, Standard



(b) Fréchet, Corrected



(c) Weibull, Standard



(d) Weibull, Corrected

Appendix A. Planning Solution, Canonical Model

Denote c_n as consumption in location n, h_n as the per-capita amount of housing in location n defined as the housing stock in location n, H_n, divided by the population in location n, L_n. Let G denote the pre-determined amount of government expenditure that needs to be funded by income taxation. The planner solves:

$$\max_{\{c_n,h_n,L_n\}_{n=1}^{N}} U$$

subject to the following constraints (Lagrange multipliers are to the left of the brackets)

$$\begin{array}{llrcl}
\text{Expected Utility} & & \lambda\left[\left(\sum_n u_n^\nu\right)^{\frac{1}{\nu}} - U\right] & = & 0 \\
\text{Resource constraint} & & (1-\alpha)\,P\left[\sum_n L_n z_n - \sum_n L_n c_n - G\right] & = & 0 \\
\text{Population:} & & \mu\left[1-\sum_n L_n\right] & = & 0 \\
\text{Utility n=1,...,N} & & \theta_n\left[A_n c_n^{1-\alpha} h_n^{\alpha} - u_n\right] & = & 0 \\
\text{Housing n=1,...,N} & & \alpha\phi_n\left[\frac{H_n}{L_n} - h_n\right] & = & 0 \\
\text{Individual optimization n=1,...,N:} & & W_n\left[\left(\frac{u_n}{U}\right)^{\nu} - L_n\right] & = & 0
\end{array}$$

First-order conditions are

$$\begin{array}{lrcl}
u_n: & 0 & = & \lambda L_n U - \theta_n u_n + W_n \nu L_n \\
c_n: & 0 & = & \theta_n u_n - P L_n c_n \\
h_n: & 0 & = & \theta_n u_n - \phi_n h_n \\
L_n: & 0 & = & (1-\alpha)\,P\,(z_n - c_n)\,L_n - \alpha\phi_n h_n - W_n L_n - \mu L_n \\
U: & 0 & = & 1-\lambda - (\nu/U)\sum_n W_n L_n
\end{array}$$

From the FOC for U we have $(\nu/U)\left(\sum_n W_n L_n\right) = 1-\lambda$. Add the Focs for u_n to get $1 = \sum_n \theta_n\,(u_n/U)$. Now add the FOCs for c_n to get $(U/P) = GDP - G$ where $GDP = \sum_n z_n L_n$.

Now start with the FOC for L_1

$$0 = (1-\alpha)\,P\,(z_1 - c_1)\,L_1 - \alpha\phi_1 h_1 - W_1 L_1 - \mu L_1$$

Note that $\phi_1 h_1 = \theta_1 u_1 = PL_1 c_1$ and insert to get

$$0 \;=\; (1-\alpha)\, PL_1 z_1 \;-\; PL_1 c_1 \;-\; W_1 L_1 \;-\; \mu L_1$$

Now use FOC for u_n

$$W_n L_n \;=\; \frac{1}{\nu}\,(\theta_n u_n) \;-\; \frac{1}{\nu}\,(\lambda L_n U) \;=\; \frac{1}{\nu}\,(PL_n c_n) \;-\; \frac{1}{\nu}\,(\lambda L_n U)$$

Insert

$$\begin{aligned} 0 &= (1-\alpha)\, PL_n z_n \;-\; PL_n c_n \;-\; \frac{1}{\nu}\,(PL_n c_n) \;+\; \frac{1}{\nu}\,(\lambda L_n U) \;-\; \mu L_n \\ &= \left[\frac{\nu\,(1-\alpha)}{1+\nu}\right] z_n \;-\; c_n \;+\; \left(\frac{\lambda-\mu/U}{1+\nu}\right)\left(\frac{U}{P}\right) \end{aligned}$$

Substituting for U/P gives

$$c_n \;=\; \left[\frac{(1-\alpha)\,\nu}{1+\nu}\right] z_n \;+\; \left(\frac{\lambda-\mu/U}{1+\nu}\right)[GDP - G] \tag{A.1}$$

Note that

$$(1+\alpha\nu)\, GDP \;=\; (\lambda-\mu/U)\,(GDP - G) \;+\; (1+\nu)\, G \tag{A.2}$$

After inserting equation (A.2) into (A.1), we get the following expression for optimal consumption in city n

$$\begin{aligned} c_n &= (1-\tau)\, z_n \;+\; T \\ \text{where} \quad \tau &= \left(\frac{1+\alpha\nu}{1+\nu}\right) \quad \text{and} \quad T \;=\; \tau \cdot GDP \;-\; G \end{aligned}$$

Appendix B. Notes on the Two-Location Model with Weibull Draws

Consider the simple model with two locations in which household i occupies city n only if $A_n c_n e_{in} > A_m c_m e_{im}$, where the variables e_{in} and e_{im} are drawn independently from a cumulative distribution function F. Previously, we considered the case where F is the Fréchet distribution (also know as the inverse Weibull distribution) with shape parameter $\nu > 0$, $F(x) = e^{-x^{-\nu}}$. Now consider the case where F is the Weibull distribution with shape parameter $k > 0$, $F(x) = 1 - e^{-x^k}$. We begin by noting that e_{in} and e_{im} being independent Weibull (shape k) means that $(e_{in}, e_{im}) = (U^{1/k}, V^{1/k})$ where U and V are standard exponential. We can now write,

$$\begin{aligned} e_{in} > te_{im} \quad \longleftrightarrow \quad & U^{1/k} > tV^{1/k} \\ & V < rU \end{aligned}$$

where $r = t^{-k}$. Since U and V are independent,

$$Pr(V < rU) = 1 - e^{-rU} \tag{B.1}$$

Note that the the probability the household chooses location 1, the expected value of (B.1), is equal to $1-(1+r)^{-1}$. Therefore $(1+r)^{-1}$ is the probability the household chooses location two, giving us

$$\begin{aligned} \frac{L_n}{L_m} &= \frac{1-(1+r)^{-1}}{(1+r)^{-1}} = \frac{1}{(1+r)^{-1}} - 1 = (1+r) - 1 = r \\ &= \left(\frac{A_n c_n}{A_m c_m}\right)^k \end{aligned}$$

where the last line uses $r = t^{-1/k}$. This implies that the elasticity of relative location choice with respect to relative consumption, $\frac{d\ln(L_n/L_m)}{d\ln(c_n/c_m)}$ is k. Recall that this elasticity is equal to ν when e_n and e_m are drawn from Fréchet. This is why we set $\nu = k$ in our numerical experiments.

Appendix C. Planning Solution, Two Location Model with Agglomeration in One Location

The planning problem for the two-location model with no location-preference draws and with agglomeration externalities in one location follows:

$$\max_{c_n,c_m,L_n,L_m} U$$

$$\begin{aligned}
\text{subject to} \quad 0 &= P\left[L_n z_n + L_m^{1+\delta} z_m - L_m c_m - L_n c_n\right] \\
0 &= \mu\left[1 - L_n - L_m\right] \\
0 &= W_n\left[A_n H_n^{\alpha} c_n^{1-\alpha} L_n^{-\alpha} - U\right] \\
0 &= W_m\left[A_m H_m^{\alpha} c_m^{1-\alpha} L_m^{-\alpha} - U\right]
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
c_n: \quad PL_n c_n &= (1-\alpha) W_n U \\
c_m: \quad PL_m c_m &= (1-\alpha) W_m U \\
L_n: \quad \mu L_n &= PL_n\left[z_n - c_n\right] - \alpha W_n U \\
L_m: \quad \mu L_m &= PL_m\left[(1+\delta) L_m^{\delta} z_m - c_m\right] - \alpha W_m U \\
U: \quad 0 &= 1 - W_n - W_m
\end{aligned}$$

From the last FOC we know $W_n + W_m = 1$. Add the FOCs for c_m and c_n and use the fact that $L_n c_n + L_m c_m = GDP$ to get

$$P \cdot GDP = (1-\alpha) U \quad \text{and thus} \quad \frac{U}{P} = \frac{GDP}{1-\alpha}$$

which implies

$$W_n = \frac{L_n c_n}{GDP} \quad \text{and} \quad W_m = \frac{L_m c_m}{GDP}$$

Now add FOCs for L_n and L_m and use resource constraint to get

$$\begin{aligned}\mu\left(L_n+L_m\right) &= P\left[z_nL_n-c_nL_n\right]+P\left[\left(1+\delta\right)L_m^{1+\delta}z_m-c_mL_m\right]-\alpha\left(W_n+W_m\right)U \\ \mu &= P\delta Y_m \ - \ \alpha U\end{aligned}$$

where $Y_m = L_m^{1+\delta}z_m$. Now derive result for optimal consumption starting with FOC for L_n

$$\begin{aligned}\mu L_n &= PL_n\left[z_n-c_n\right] \ - \ \alpha W_nU \\ \left(P\delta Y_m-\alpha U\right)L_n &= PL_n\left[z_n-c_n\right] \ - \ \alpha W_nU \\ \left(1-\alpha\right)\delta Y_mL_n-\alpha\cdot GDP\cdot L_n &= \left(1-\alpha\right)z_nL_n \ - \ \left(1-\alpha\right)c_nL_n \ - \ \alpha c_nL_n \\ c_nL_n &= \left(1-\alpha\right)\left(z_n-\delta Y_m\right)L_n \ + \ \alpha\cdot GDP\cdot L_n \\ c_n &= \left(1-\alpha\right)\left(z_n-\delta Y_m\right) \ + \ \alpha\cdot GDP\end{aligned}$$

after similar math for c_m we get the result

$$c_m = \left(1-\alpha\right)\left[\left(1+\delta\right)L_m^{\delta}z_m-\delta Y_m\right] \ + \ \alpha\cdot GDP$$